

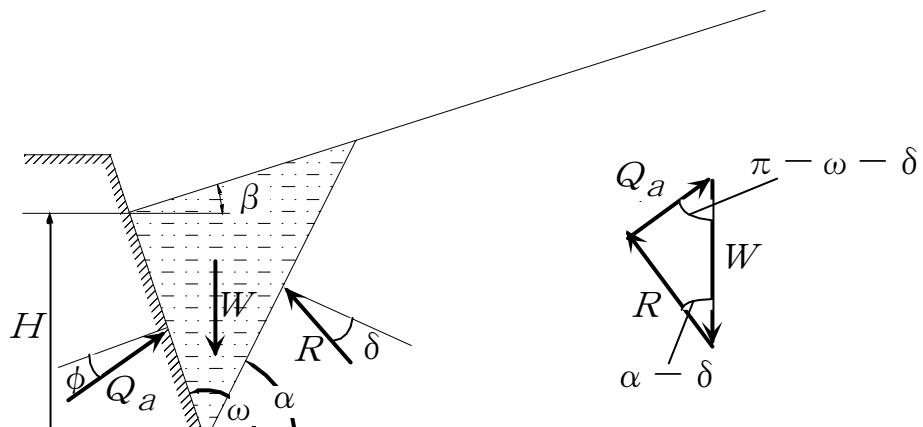
Coulomb 土圧の導入

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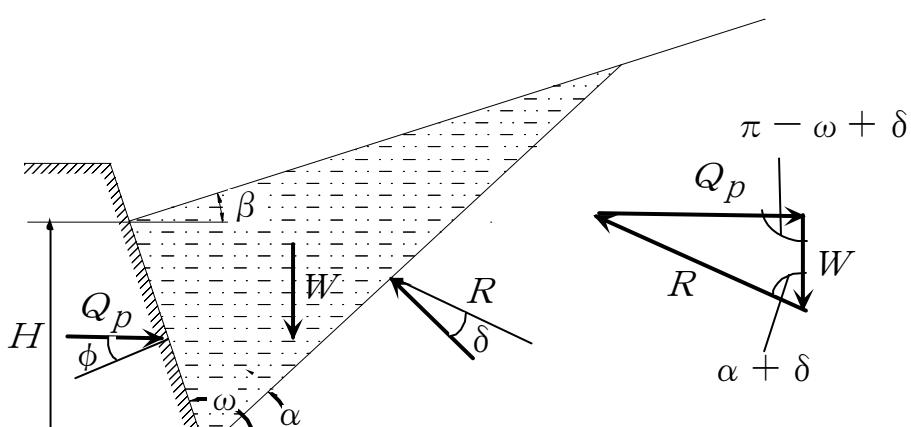
Coulomb 土圧は次式で与えられる。

$$\begin{aligned} \left(\frac{Q_a}{Q_p} \right) &= \frac{1}{2} \gamma_t H^2 \left(\frac{K_a}{K_p} \right) \\ \left(\frac{K_a}{K_p} \right) &= \left[\frac{\sin(\omega \mp \phi)}{\sin \omega \left\{ \sqrt{\sin(\omega \pm \delta)} \pm \sqrt{\frac{\sin(\phi + \delta) \sin(\phi \mp \beta)}{\sin(\omega - \beta)}} \right\}} \right] \end{aligned}$$

この土圧係数 $K_a K_p$ を以下誘導する。



(a) 主働土圧



(b) 受動土圧

Fig. 1

破壊領域の重量Wの計算 (Fig.2)

$$W = \frac{1}{2} \gamma_t \cdot \overline{AB} \cdot \overline{BC} \cdot \sin(\omega - \alpha) \dots (1)$$

$$\overline{AB} = \frac{H}{\sin \omega} \dots (2)$$

$$\frac{\overline{AB}}{\sin(\angle C)} = \frac{\overline{BC}}{\sin(\angle A)} \dots \text{(正弦定理)}$$

$$\begin{aligned} \overline{BC} &= \frac{H}{\sin \omega} \cdot \frac{\sin(\pi - \omega + \beta)}{\sin(\alpha - \beta)} \\ &= \frac{H}{\sin \omega} \cdot \frac{\sin(\omega - \beta)}{\sin(\alpha - \beta)} \dots (3) \end{aligned}$$

(2)、(3)を(1)に代入

$$\begin{aligned} W &= \frac{1}{2} \gamma_t H^2 \frac{\sin(\omega - \beta) \sin(\omega - \alpha)}{\sin^2 \omega \sin(\alpha - \beta)} \\ &= \frac{1}{2} \gamma_t H^2 \frac{\sin(\omega - \beta) \sin((\omega - \beta) - (\alpha - \beta))}{\sin^2 \omega \sin(\alpha - \beta)} \\ &= \frac{1}{2} \gamma_t H^2 \frac{\sin(\omega - \beta) (\sin(\omega - \beta) \cos(\alpha - \beta) - \cos(\omega - \beta) \sin(\alpha - \beta))}{\sin^2 \omega \sin(\alpha - \beta)} \\ &= \frac{1}{2} \gamma_t H^2 \frac{\sin^2(\omega - \beta)}{\sin^2 \omega} \{ \cot(\alpha - \beta) - \cot(\omega - \beta) \} \dots (4) \end{aligned}$$

力の釣り合い (Fig.3)

$$\frac{W}{\sin(\omega + \delta + \phi - \alpha)} = \frac{Q}{\sin(\alpha - \phi)} \dots \text{(正弦定理)}$$

$$\begin{aligned} Q &= W \frac{\sin(\alpha - \phi)}{\sin(\omega + \delta + \phi - \alpha)} \\ &= W \frac{\sin((\alpha - \beta) - (\phi - \beta))}{\sin((\omega + \delta + \phi - \beta) - (\alpha - \beta))} \\ &= W \frac{\sin(\alpha - \beta) \cos(\phi - \beta) - \cos(\alpha - \beta) \sin(\phi - \beta)}{\sin(\omega + \delta + \phi - \beta) \cos(\alpha - \beta) - \cos(\omega + \delta + \phi - \beta) \sin(\alpha - \beta)} \\ &= W \frac{\cos(\phi - \beta) - \sin(\phi - \beta) \cot(\alpha - \beta)}{\sin(\omega + \delta + \phi - \beta) \cot(\alpha - \beta) - \cos(\omega + \delta + \phi - \beta)} \dots (5) \end{aligned}$$

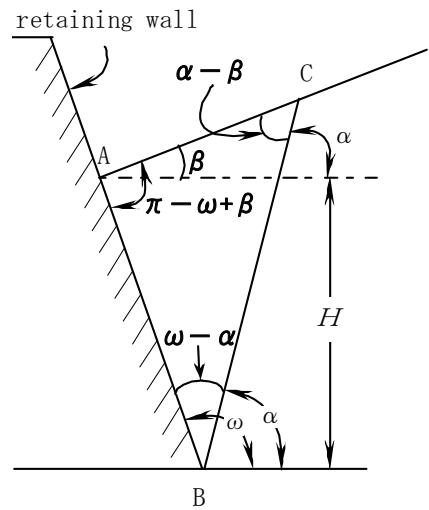


Fig. 2

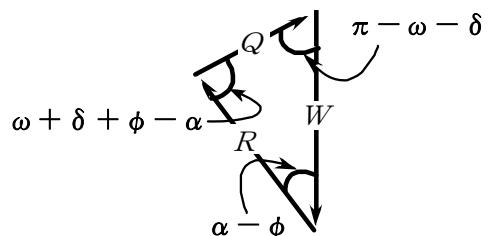


Fig. 3

(4) を (5) に代入

$$Q = \frac{1}{2} \gamma_t \frac{\sin^2(\omega - \beta)}{\sin^2 \omega} \{ \cot(\alpha - \beta) - \cot(\omega - \beta) \} \frac{\cos(\phi - \beta) - \sin(\phi - \beta) \cot(\alpha - \beta)}{\sin(\omega + \delta + \phi - \beta) \cot(\alpha - \beta) - \cos(\omega + \delta + \phi - \beta)} \dots \quad (6)$$

$$\left\{ \begin{array}{l} x = \cot(\alpha - \beta) \\ \mu = \omega + \delta + \phi - \beta \\ \phi' = \phi - \beta \\ \omega' = \omega - \beta \end{array} \right\} \dots \quad (7) \quad \left\{ \begin{array}{l} \mu - \phi' = \omega + \delta \\ \mu - \omega' = \delta + \phi \end{array} \right\} \dots \quad (7) ,$$

(7) より

$$Q = \frac{1}{2} \gamma_t \frac{\sin^2 \omega'}{\sin^2 \omega} \{ x - \cot \omega' \} \frac{\cos \phi' - x \sin \phi'}{x \sin \mu - \cos \mu} \dots \quad (8)$$

(8) より土圧合力 Q は $x = \cot(\alpha - \beta)$ の関数である。

主働土圧 Q_a は土圧合力 Q が最小となる場合であるので、 $\frac{\partial Q}{\partial x} = 0$ を満たす x_a を決め、そのときの土圧を求めればよい。

$$\begin{aligned} \frac{\partial Q}{\partial x} &= \frac{\partial}{\partial x} \left[\frac{1}{2} \gamma_t \frac{\sin^2 \omega'}{\sin^2 \omega} \{ x - \cot \omega' \} \frac{\cos \phi' - x \sin \phi'}{x \sin \mu - \cos \mu} \right] = 0 \\ &\Leftrightarrow \frac{\partial}{\partial x} \left[(x - \cot \omega') \frac{\cos \phi' - x \sin \phi'}{x \sin \mu - \cos \mu} \right] = 0 \\ &\Leftrightarrow \frac{\cos \phi' - x \sin \phi'}{x \sin \mu - \cos \mu} + (x - \cot \omega') \frac{\partial}{\partial x} \left[\frac{\cos \phi' - x \sin \phi'}{x \sin \mu - \cos \mu} \right] = 0 \\ &\Leftrightarrow \frac{\cos \phi' - x \sin \phi'}{x \sin \mu - \cos \mu} + (x - \cot \omega') \left\{ \frac{(-\sin \phi')(x \sin \mu - \cos \mu) - \sin \mu (\cos \phi' - x \sin \phi')}{(x \sin \mu - \cos \mu)^2} \right\} = 0 \\ &\Leftrightarrow \frac{\cos \phi' - x \sin \phi'}{x \sin \mu - \cos \mu} + \frac{(x - \cot \omega')(-\sin \phi')}{x \sin \mu - \cos \mu} + \frac{-\sin \mu (x - \cot \omega')(\cos \phi' - x \sin \phi')}{(x \sin \mu - \cos \mu)^2} = 0 \\ &\Leftrightarrow (x \sin \mu - \cos \mu)(\cos \phi' - x \sin \phi') + (x - \cot \omega')(-\sin \phi')(x \sin \mu - \cos \mu) + (-\sin \mu)(x - \cot \omega')(\cos \phi' - x \sin \phi') = 0 \end{aligned}$$

展開して整理

$$(-\sin \mu \sin \phi')x^2 + 2(\cos \mu \sin \phi')x - (\cos \mu \cos \phi' + \cot \omega' \cos \mu \sin \phi' - \cot \omega' \sin \mu \cos \phi') = 0 \dots \quad (9)$$

(9) の正の解

$$x_a = \frac{1}{(-\sin \mu \sin \phi')} \left\{ -\cos \mu \sin \phi' + \sqrt{(\cos \mu \sin \phi')^2 + (-\sin \mu \sin \phi')(\cos \mu \cos \phi' + \cos \omega \cos \mu \sin \phi' - \cot \omega' \sin \mu \cos \phi')} \right\} \dots \quad (10)$$

(10) の $\sqrt{}$ 内

$$\begin{aligned}
 & (\cos \mu \sin \phi')^2 + (-\sin \mu \sin \phi')(\cos \mu \cos \phi' + \cos \omega \cos \mu \sin \phi' - \cot \omega' \sin \mu \cos \phi') \\
 &= \cos^2 \mu \sin^2 \phi' - \sin \mu \sin \phi' \cos \mu \cos \phi' - \sin \mu \sin^2 \phi' \cos \mu \cot \omega' + \sin \mu \sin \phi' \sin \mu \cos \phi' \cot \omega' \\
 &= \frac{\sin \phi'}{\sin \omega'} (\cos^2 \mu \sin \phi' \sin \omega' - \sin \mu \cos \mu \cos \phi' \sin \omega' - \sin \mu \sin \phi' \cos \mu \cos \omega' + \sin \mu \sin \mu \cos \phi' \cos \omega') \\
 &= \frac{\sin \phi'}{\sin \omega'} (\cos \mu \sin \phi' - \sin \mu \cos \phi') (\cos \mu \sin \omega' - \sin \mu \cos \omega') \\
 &= \frac{\sin \phi'}{\sin \omega'} \sin(\phi' - \mu) \sin(\omega' - \mu) \\
 &= \frac{\sin \phi'}{\sin \omega'} \sin(\omega + \delta) \sin(\delta + \phi) \quad \leftarrow (7) \text{ より}
 \end{aligned}$$

(10) に戻って

$$\begin{aligned}
 x_a &= \frac{1}{(-\sin \mu \sin \phi')} \left\{ -\cos \mu \sin \phi' + \sqrt{\frac{\sin \phi'}{\sin \omega'} \sin(\omega + \delta) \sin(\delta + \phi)} \right\} \\
 x_a &= \cot \mu - \frac{1}{\sin \mu} \sqrt{\frac{\sin(\omega + \delta) \sin(\delta + \phi)}{\sin \phi' \sin \omega'}} \quad \dots \dots (11)
 \end{aligned}$$

$$x = x_a \text{ と } Q = Q_a \quad (8) \text{ より}$$

$$\begin{cases} Q_a = \frac{1}{2} \gamma_t K_a \\ K_a = \frac{\sin^2 \omega'}{\sin^2 \omega} \{x_a - \cot \omega'\} \frac{\cos \phi' - x_a \sin \phi'}{x_a \sin \mu - \cos \mu} \end{cases} \quad \dots \dots (12)$$

↑
Z

$$Z = x_a \sin \mu - \cos \mu = \left(\cot \mu - \frac{1}{\sin \mu} \sqrt{\frac{\sin(\omega + \delta) \sin(\delta + \phi)}{\sin \phi' \sin \omega'}} \right) \sin \mu - \cos \mu = -\sqrt{\frac{\sin(\omega + \delta) \sin(\delta + \phi)}{\sin \phi' \sin \omega'}}$$

$$\text{と置く} \quad \dots \dots (13)$$

$$\begin{aligned}
 K_a &= \frac{\sin^2 \omega'}{\sin^2 \omega} (\cot \omega' - x_a) \frac{1}{Z} (\cos \phi' - x_a \sin \phi') \\
 &= \frac{\sin^2 \omega'}{\sin^2 \omega} \frac{\cos \phi'}{Z} (\cot \omega' - x_a) (1 - x_a \tan \phi') \quad \dots \dots (14)
 \end{aligned}$$

$$(13) \rightarrow x_a = \frac{Z + \cos \mu}{\sin \mu} \quad \dots \dots (15)$$

$$\begin{aligned}
 (14) \rightarrow K_a &= \frac{\sin^2 \omega'}{\sin^2 \omega} \frac{\cos \phi'}{Z} \left(\cot \omega' - x_a \cot \omega' \tan \phi' - x_a + x_a^2 \tan \phi' \right) \\
 &= \frac{\sin^2 \omega'}{\sin^2 \omega} \frac{\cos \phi'}{Z} \left(\cot \omega' - \frac{Z + \cos \mu}{\sin \mu} \cot \omega' \tan \phi' - \frac{Z + \cos \mu}{\sin \mu} + \frac{(Z + \cos \mu)^2}{\sin^2 \mu} \tan \phi' \right) \\
 &= \frac{\sin^2 \omega'}{\sin^2 \omega} \frac{\cos \phi'}{Z \sin^2 \mu} \left(\sin^2 \mu \cot \omega' - (Z + \cos \mu) \sin \mu \cot \omega' \tan \phi' - (Z + \cos \mu) \sin \mu + (Z + \cos \mu)^2 \tan \phi' \right) \dots \dots \\
 (16)
 \end{aligned}$$

(16) のカッコ内を A とおくと

$$\begin{aligned}
 A &= \sin^2 \mu \cot \omega' - (Z + \cos \mu) \sin \mu \cot \omega' \tan \phi' - (Z + \cos \mu) \sin \mu + (Z + \cos \mu)^2 \tan \phi' \\
 &= \tan \phi' (Z^2 + 2 \cdot Z \cos \mu + \cos^2 \mu) + \tan \phi' \cot \omega' (-Z \sin \mu - \sin \mu \cos \mu) - Z \sin \mu - \sin \mu \cos \mu + \sin^2 \mu \cot \omega' \\
 &= Z^2 \tan \phi' + 2 \cdot Z \cos \mu \tan \phi' + \cos^2 \mu \tan \phi' - Z \sin \mu \tan \phi' \cot \omega' - \sin \mu \cos \mu \tan \phi' \cot \omega' - Z \sin \mu - \sin \mu \cos \mu + \sin^2 \mu \cot \omega' \\
 A_1 &\quad A_1 \quad A_2
 \end{aligned}$$

$\dots \dots (17)$

$$\begin{aligned}
 A_1 &= \cos^2 \mu \tan \phi' - \sin \mu \cos \mu \tan \phi' \cot \omega' \\
 &= \cos \mu \tan \phi' (\cos \mu - \sin \mu \cot \omega') \\
 &= \cos \mu \tan \phi' \frac{\sin \omega' \cos \mu - \cos \omega' \sin \mu}{\sin \omega'} \\
 &= \cos \mu \tan \phi' \frac{\sin(\omega' - \mu)}{\sin \omega'} \\
 &= -\cos \mu \tan \phi' \frac{\sin(\delta + \phi)}{\sin \omega'} \quad \dots \dots (18)
 \end{aligned}$$

$$\begin{aligned}
 A_2 &= -\sin \mu \cos \mu + \sin^2 \mu \cot \omega' \\
 &= -\sin \mu (\cos \mu - \sin \mu \cot \omega') \\
 &= -\sin \mu \frac{\sin \omega' \cos \mu - \cos \omega' \sin \mu}{\sin \omega'} \\
 &= -\sin \mu \frac{\sin(\omega' - \mu)}{\sin \omega'} \\
 &= \sin \mu \frac{\sin(\delta + \phi)}{\sin \omega'} \quad \dots \dots (19)
 \end{aligned}$$

(18)、(19) より

$$\begin{aligned}
 A_1 + A_2 &= -\cos \mu \tan \phi' \frac{\sin(\delta + \phi)}{\sin \omega'} + \sin \mu \frac{\sin(\delta + \phi)}{\sin \omega'} \\
 &= -\frac{\sin(\delta + \phi)}{\sin \omega'} (\cos \mu \tan \phi' - \sin \mu) \\
 &= -\frac{\sin(\delta + \phi)}{\sin \omega'} \cdot \frac{\cos \mu \sin \phi' - \sin \mu \cos \phi'}{\cos \phi'} \\
 &= -\frac{\sin(\delta + \phi)}{\sin \omega'} \cdot \frac{\sin(\phi' - \mu)}{\cos \phi'} \\
 &= \frac{\sin(\delta + \phi) \sin(\omega + \delta)}{\sin \omega' \cos \phi'} \quad \cdots \cdots (20)
 \end{aligned}$$

$$(20) = A_1 + A_2 = \frac{\sin(\delta + \phi) \sin(\omega + \delta)}{\sin \omega' \sin \phi'} \cdot \frac{\sin \phi'}{\cos \phi'} = Z^2 \tan \phi' \quad \cdots \cdots (21) \quad \leftarrow (13) \text{ より}$$

以上より、(16) を整理すると

$$K_a = \frac{\sin^2 \omega'}{\sin^2 \omega} \frac{\cos \phi'}{Z \sin^2 \mu} (2 \cdot Z^2 \tan \phi' + 2 \cdot Z \cos \mu \tan \phi' - Z \sin \mu \tan \phi' \cot \omega' - Z \sin \mu)$$

$$K_a = \frac{\sin^2 \omega'}{\sin^2 \omega} \frac{\cos \phi'}{\sin^2 \mu} (2 \cdot Z \tan \phi' + 2 \cos \mu \tan \phi' - \sin \mu \tan \phi' \cot \omega' - \sin \mu) \quad \cdots \cdots (22)$$

(22) のカッコ内を B と置くと

$$\begin{aligned}
 B &= 2 \cdot Z \tan \phi' + 2 \cos \mu \tan \phi' - \sin \mu \tan \phi' \cot \omega' - \sin \mu \\
 &= 2 \cdot Z \tan \phi' + (\cos \mu \tan \phi' - \sin \mu \tan \phi' \cot \omega') + (\cos \mu \tan \phi' - \sin \mu) \quad \cdots \cdots (23)
 \end{aligned}$$

$$B_1 = \cos \mu \tan \phi' - \sin \mu \tan \phi' \cot \omega'$$

$$= \tan \phi' (\cos \mu - \sin \mu \cot \omega')$$

$$= \tan \phi' \frac{\cos \mu \sin \omega' - \sin \mu \cos \omega'}{\sin \omega'}$$

$$= \tan \phi' \frac{\sin(\omega' - \mu)}{\sin \omega'}$$

$$= -\tan \phi' \frac{\sin(\delta + \phi)}{\sin \omega'} \quad \cdots \cdots (24)$$

$$\begin{aligned}
B_2 &= \cos \mu \tan \phi' - \sin \mu \\
&= \frac{\cos \mu \sin \phi' - \cos \phi' \sin \mu}{\cos \phi'} \\
&= \frac{\sin(\phi' - \mu)}{\cos \phi'} \\
&= -\frac{\sin(\omega + \delta)}{\cos \phi'} \quad \cdots \cdots (25)
\end{aligned}$$

(24)、(25) → (22)

$$\begin{aligned}
K_a &= \frac{\sin^2 \omega'}{\sin^2 \omega} \frac{\cos \phi'}{\sin^2 \mu} \left(2 \cdot Z \tan \phi' - \tan \phi' \frac{\sin(\delta + \phi)}{\sin \omega'} - \frac{\sin(\omega + \delta)}{\cos \phi'} \right) \\
&= -\frac{\sin^2 \omega'}{\sin^2 \omega} \frac{\cos \phi'}{\sin^2 \mu} \left(\frac{\tan \phi'}{\sin \omega'} \sin(\delta + \phi) + 2 \tan \phi' \sqrt{\frac{\sin(\omega + \delta) \sin(\delta + \phi)}{\sin \phi' \sin \omega'}} + \frac{\sin(\omega + \delta)}{\cos \phi'} \right)
\end{aligned}$$

$$\begin{aligned}
&= -\frac{\sin^2 \omega'}{\sin^2 \omega} \frac{\cos \phi'}{\sin^2 \mu} \left(\sqrt{\frac{\tan \phi'}{\sin \omega'} \sin(\delta + \phi)} + \sqrt{\frac{\sin(\omega + \delta)}{\cos \phi'}} \right)^2 \\
&= -\frac{\sin^2 \omega'}{\sin^2 \omega} \frac{\cos \phi'}{\sin^2 \mu} \frac{\left(\frac{\tan \phi'}{\sin \omega'} \sin(\delta + \phi) - \frac{\sin(\omega + \delta)}{\cos \phi'} \right)^2}{\left(\sqrt{\frac{\tan \phi'}{\sin \omega'} \sin(\delta + \phi)} - \sqrt{\frac{\sin(\omega + \delta)}{\cos \phi'}} \right)^2}
\end{aligned}$$

$$\begin{aligned}
&(\sqrt{x} + \sqrt{y})^2 \\
&= x + 2\sqrt{x}\sqrt{y} + y \\
&= \frac{(x+y+2\sqrt{x}\sqrt{y})(x+y-2\sqrt{x}\sqrt{y})}{(x-2\sqrt{x}\sqrt{y}+y)} \\
&= \frac{(x+y)^2 - (2\sqrt{x}\sqrt{y})^2}{(x-2\sqrt{x}\sqrt{y}+y)} \\
&= \frac{(x-y)^2}{(\sqrt{x}-\sqrt{y})^2}
\end{aligned}$$

$$\begin{aligned}
&= -\frac{\sin^2 \omega'}{\sin^2 \omega} \frac{\cos \phi'}{\sin^2 \mu} \frac{1}{(\sin^2 \omega' \cos^2 \phi')} \frac{\left(\sin \phi'^2 \sin^2(\delta + \phi) - 2 \cdot \sin \omega' \sin \phi' \sin(\delta + \phi) \sin(\omega + \delta) + \sin^2 \omega' \sin^2(\omega + \delta) \right)}{\left(\frac{\tan \phi'}{\sin \omega'} \sin(\delta + \phi) - 2 \cdot \sqrt{\frac{\sin(\delta + \phi) \sin(\omega + \delta)}{\sin \omega' \cos \phi'}} \tan \phi' + \frac{\sin(\omega + \delta)}{\cos \phi'} \right)^2} \\
&= -\frac{(\sin \phi' \sin(\delta + \phi) - \sin \omega' \sin(\omega + \delta))^2}{\sin^2 \omega \cdot \sin^2 \mu \left(\sqrt{\sin(\omega + \delta)} - \sqrt{\frac{\sin \phi'}{\sin \omega'} \sin(\delta + \phi)} \right)^2} \quad \cdots \cdots (26)
\end{aligned}$$

ここで (26) の分子を以下のように置く

$$[C]^2 = (\sin \phi' \sin(\delta + \phi) - \sin \omega' \sin(\omega + \delta))^2$$

$$\begin{aligned}
C &= \sin \phi' \sin(\delta + \phi) - \sin \omega' \sin(\omega + \delta) \\
&= \frac{1}{2}(-\cos(\delta + \phi + \phi') + \cos(\delta + \phi - \phi') + \cos(\omega + \delta + \omega') - \cos(\omega + \delta - \omega')) \\
&= \frac{1}{2}(-\cos(\delta + 2\phi - \beta) + \cos(\delta + \beta) + \cos(2\omega + \delta - \beta) - \cos(\delta + \beta)) \\
&= \frac{1}{2}(\cos(2\omega + \delta - \beta) - \cos(\delta + 2\phi - \beta)) \\
&= -\left(\sin \frac{(2\omega + \delta - \beta) + (\delta + 2\phi - \beta)}{2} \sin \frac{(2\omega + \delta - \beta) - (\delta + 2\phi - \beta)}{2}\right) \\
&= -\sin(\omega + \delta + \phi - \beta) \cdot \sin(\omega - \phi) \\
&= -\sin \mu \cdot \sin(\omega - \phi) \quad \cdots \quad (27)
\end{aligned}$$

(27) → (26)

$$\begin{aligned}
K_a &= \frac{(\sin \mu \cdot \sin(\omega - \phi))^2}{\sin^2 \omega \cdot \sin^2 \mu \left(\sqrt{\sin(\omega + \delta)} - \sqrt{\frac{\sin \phi'}{\sin \omega'} \sin(\delta + \phi)} \right)^2} \\
&= \frac{\sin^2(\omega - \phi)}{\sin^2 \omega \cdot \left(\sqrt{\sin(\omega + \delta)} - \sqrt{\frac{\sin(\phi - \beta) \sin(\delta + \phi)}{\sin \omega'}} \right)^2} \\
&= \left[\frac{\sin(\omega - \phi)}{\sin \omega \cdot \left(\sqrt{\sin(\omega + \delta)} - \sqrt{\frac{\sin(\phi - \beta) \sin(\delta + \phi)}{\sin \omega'}} \right)} \right]^2
\end{aligned}$$

以上より主働土圧について Coulomb 土圧が導かれた。

受動土圧は δ 、 ϕ の符号が逆の場合であり、その導入は同様であり省略する。

以上