Coulomb 土圧の導入

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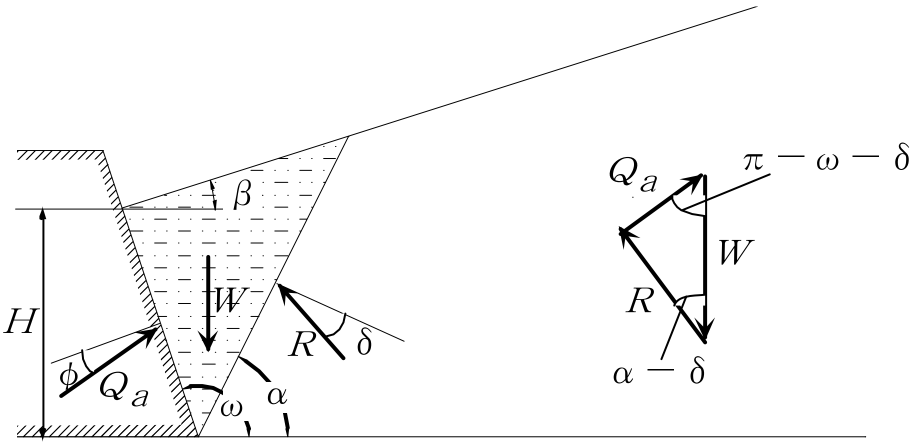
Coulomb 土圧は次式で与えられる。

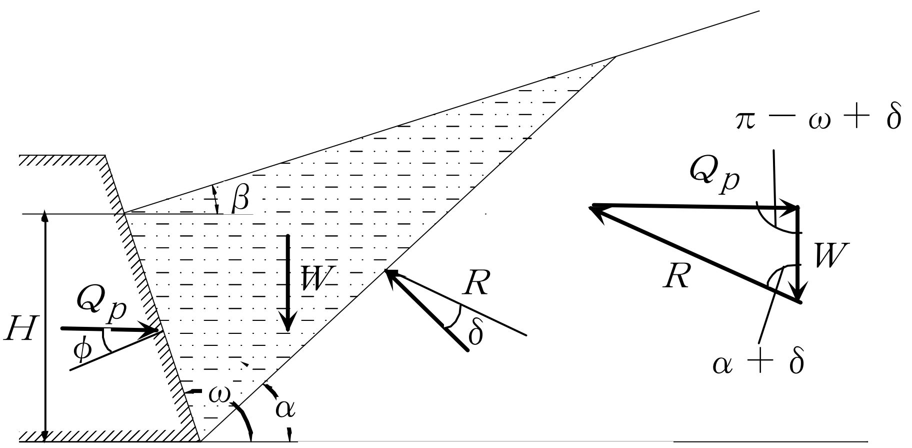
$$\begin{pmatrix} Q_a \\ Q_p \end{pmatrix} = \frac{1}{2}\gamma_t H^2 \begin{pmatrix} K_a \\ K_p \end{pmatrix}$$

$$\begin{pmatrix} K_a \\ K_p \end{pmatrix} = \left[\frac{\sin(\omega \mp \phi)}{\sin\omega \left\{ \sqrt{\sin(\omega \pm \delta)} \pm \sqrt{\dfrac{\sin(\phi+\delta)\sin(\phi \mp \beta)}{\sin(\omega-\beta)}} \right\}} \right]$$

この土圧係数 K_a, K_p を以下誘導する。



(a)主働土圧



(b)受動土圧

Fig. 1

破壊領域の重量Wの計算（**Fig.2**）

$$W=\frac{1}{2}\gamma_t\cdot\overline{\mathrm{AB}}\cdot\overline{\mathrm{BC}}\cdot\sin(\omega-\alpha)\quad\cdots(\mathbf{1})$$

$$\overline{\mathrm{AB}}=\frac{H}{\sin\omega}\quad\cdots(\mathbf{2})$$

$$\frac{\overline{\mathrm{AB}}}{\sin(\angle\mathrm{C})}=\frac{\overline{\mathrm{BC}}}{\sin(\angle\mathrm{A})}\quad\cdots(\text{正弦定理})$$

$$\overline{\mathrm{BC}}=\frac{H}{\sin\omega}\cdot\frac{\sin(\pi-\omega+\beta)}{\sin(\alpha-\beta)}$$

$$=\frac{H}{\sin\omega}\cdot\frac{\sin(\omega-\beta)}{\sin(\alpha-\beta)}\quad\cdots(\mathbf{3})$$

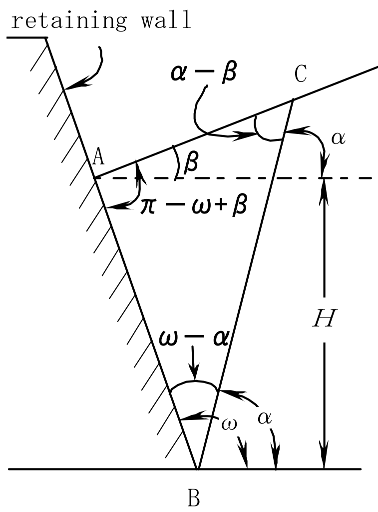


Fig. 2

（２）、（３）を（１）に代入

$$W=\frac{1}{2}\gamma_t H^2\frac{\sin(\omega-\beta)\sin(\omega-\alpha)}{\sin^2\omega\sin(\alpha-\beta)}$$

$$=\frac{1}{2}\gamma_t H^2\frac{\sin(\omega-\beta)\sin\{(\omega-\beta)-(\alpha-\beta)\}}{\sin^2\omega\sin(\alpha-\beta)}$$

$$=\frac{1}{2}\gamma_t H^2\frac{\sin(\omega-\beta)\{\sin(\omega-\beta)\cos(\alpha-\beta)-\cos(\omega-\beta)\sin(\alpha-\beta)\}}{\sin^2\omega\sin(\alpha-\beta)}$$

$$=\frac{1}{2}\gamma_t H^2\frac{\sin^2(\omega-\beta)}{\sin^2\omega}\{\cot(\alpha-\beta)-\cot(\omega-\beta)\}\quad\cdots(4)$$

力の釣り合い（**Fig.3**）

$$\frac{W}{\sin(\omega+\delta+\phi-\alpha)}=\frac{Q}{\sin(\alpha-\phi)}\quad\cdots(\text{正弦定理})$$

$$Q=W\frac{\sin(\alpha-\phi)}{\sin(\omega+\delta+\phi-\alpha)}$$

$$=W\frac{\sin\{(\alpha-\beta)-(\phi-\beta)\}}{\sin\{(\omega+\delta+\phi-\beta)-(\alpha-\beta)\}}$$

$$=W\frac{\sin(\alpha-\beta)\cos(\phi-\beta)-\cos(\alpha-\beta)\sin(\phi-\beta)}{\sin(\omega+\delta+\phi-\beta)\cos(\alpha-\beta)-\cos(\omega+\delta+\phi-\beta)\sin(\alpha-\beta)}$$

$$=W\frac{\cos(\phi-\beta)-\sin(\phi-\beta)\cot(\alpha-\beta)}{\sin(\omega+\delta+\phi-\beta)\cot(\alpha-\beta)-\cos(\omega+\delta+\phi-\beta)}\quad\cdots(\mathbf{5})$$

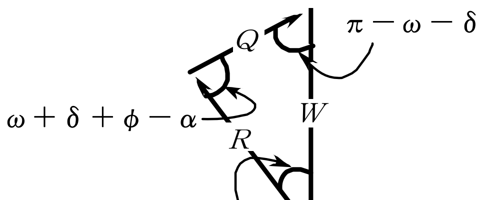


Fig. 3

（４）を（５）に代入

$$Q=\frac{1}{2}\gamma_t\frac{\sin^2(\omega-\beta)}{\sin^2\omega}\{\cot(\alpha-\beta)-\cot(\omega-\beta)\}\frac{\cos(\phi-\beta)-\sin(\phi-\beta)\cot(\alpha-\beta)}{\sin(\omega+\delta+\phi-\beta)\cot(\alpha-\beta)-\cos(\omega+\delta+\phi-\beta)} \quad \cdots (6)$$

$$\begin{Bmatrix} x=\cot(\alpha-\beta) \\ \mu=\omega+\delta+\phi-\beta \\ \phi'=\phi-\beta \\ \omega'=\omega-\beta \end{Bmatrix} \cdots (7) \qquad \begin{Bmatrix} \mu-\phi'=\omega+\delta \\ \mu-\omega'=\delta+\phi \end{Bmatrix} \cdots (7)'$$

（７）より

$$Q=\frac{1}{2}\gamma_t\frac{\sin^2\omega'}{\sin^2\omega}\{x-\cot\omega'\}\frac{\cos\phi'-x\sin\phi'}{x\sin\mu-\cos\mu} \quad \cdots (8)$$

（８）より土圧合力 Q は $x=\cot(\alpha-\beta)$ の関数である。

主働土圧 Q_a は土圧合力 Q が最小となる場合であるので、$\partial Q/\partial x=0$ を満たす x_a を決め、そのときの土圧を求めればよい。

$$\frac{\partial Q}{\partial x}=\frac{\partial}{\partial x}\left[\frac{1}{2}\gamma_t\frac{\sin^2\omega'}{\sin^2\omega}\{x-\cot\omega'\}\frac{\cos\phi'-x\sin\phi'}{x\sin\mu-\cos\mu}\right]=0$$

$$\Leftrightarrow \frac{\partial}{\partial x}\left[(x-\cot\omega')\frac{\cos\phi'-x\sin\phi'}{x\sin\mu-\cos\mu}\right]=0$$

$$\Leftrightarrow \frac{\cos\phi'-x\sin\phi'}{x\sin\mu-\cos\mu}+(x-\cot\omega')\frac{\partial}{\partial x}\left[\frac{\cos\phi'-x\sin\phi'}{x\sin\mu-\cos\mu}\right]=0$$

$$\Leftrightarrow \frac{\cos\phi'-x\sin\phi'}{x\sin\mu-\cos\mu}+(x-\cot\omega')\left\{\frac{(-\sin\phi')(x\sin\mu-\cos\mu)-\sin\mu(\cos\phi'-x\sin\phi')}{(x\sin\mu-\cos\mu)^2}\right\}=0$$

$$\Leftrightarrow \frac{\cos\phi'-x\sin\phi'}{x\sin\mu-\cos\mu}+\frac{(x-\cot\omega')(-\sin\phi')}{x\sin\mu-\cos\mu}+\frac{-\sin\mu(x-\cot\omega')(\cos\phi'-x\sin\phi')}{(x\sin\mu-\cos\mu)^2}=0$$

$$\Leftrightarrow (x\sin\mu-\cos\mu)(\cos\phi'-x\sin\phi')+(x-\cot\omega')(-\sin\phi')(x\sin\mu-\cos\mu)+(-\sin\mu)(x-\cot\omega')(\cos\phi'-x\sin\phi')=0$$

展開して整理

$$(-\sin\mu\sin\phi')x^2+2(\cos\mu\sin\phi')x-(\cos\mu\cos\phi'+\cot\omega'\cos\mu\sin\phi'-\cot\omega'\sin\mu\cos\phi')=0 \quad \cdots (9)$$

（９）の正の解

$$x_a=\frac{1}{(-\sin\mu\sin\phi')}\left\{-\cos\mu\sin\phi'+\sqrt{(\cos\mu\sin\phi')^2+(-\sin\mu\sin\phi')(\cos\mu\cos\phi'+\cos\omega\cos\mu\sin\phi'-\cot\omega'\sin\mu\cos\phi')}\right\}$$

$$\cdots (10)$$

（10）の$\sqrt{\ }$内

$$(\cos\mu\sin\phi')^2+(-\sin\mu\sin\phi')(\cos\mu\cos\phi'+\cos\omega\cos\mu\sin\phi'-\cot\omega'\sin\mu\cos\phi')$$

$$=\cos^2\mu\sin^2\phi'-\sin\mu\sin\phi'\cos\mu\cos\phi'-\sin\mu\sin^2\phi'\cos\mu\cot\omega'+\sin\mu\sin\phi'\sin\mu\cos\phi'\cot\omega'$$

$$=\frac{\sin\phi'}{\sin\omega'}\left(\cos^2\mu\sin\phi'\sin\omega'-\sin\mu\cos\mu\cos\phi'\sin\omega'-\sin\mu\sin\phi'\cos\mu\cos\omega'+\sin\mu\sin\mu\cos\phi'\cos\omega'\right)$$

$$=\frac{\sin\phi'}{\sin\omega'}(\cos\mu\sin\phi'-\sin\mu\cos')(\cos\mu\sin\omega'-\sin\mu\cos\omega')$$

$$=\frac{\sin\phi'}{\sin\omega'}\sin(\phi'-\mu)\sin(\omega'-\mu)$$

$$=\frac{\sin\phi'}{\sin\omega'}\sin(\omega+\delta)\sin(\delta+\phi)\qquad\leftarrow\text{（７）'より}$$

（10）に戻って

$$x_a=\frac{1}{(-\sin\mu\sin\phi')}\left\{-\cos\mu\sin\phi'+\sqrt{\frac{\sin\phi'}{\sin\omega'}\sin(\omega+\delta)\sin(\delta+\phi)}\right\}$$

$$x_a=\cot\mu-\frac{1}{\sin\mu}\sqrt{\frac{\sin(\omega+\delta)\sin(\delta+\phi)}{\sin\phi'\sin\omega'}}\quad\cdot\cdot\cdot\ \mathbf{(11)}$$

$x=x_a$ のとき $Q=Q_a$　　（８）より

$$\left\{\begin{aligned}Q_a&=\frac{1}{2}\gamma_t K_a\\ K_a&=\frac{\sin^2\omega'}{\sin^2\omega}\{x_a-\cot\omega'\}\frac{\cos\phi'-x_a\sin\phi'}{x_a\sin\mu-\cos\mu}\end{aligned}\right\}\quad\cdot\cdot\cdot\ \mathbf{(12)}$$

Z

$$Z=x_a\sin\mu-\cos\mu=\left(\cot\mu-\frac{1}{\sin\mu}\sqrt{\frac{\sin(\omega+\delta)\sin(\delta+\phi)}{\sin\phi'\sin\omega'}}\right)\sin\mu-\cos\mu=-\sqrt{\frac{\sin(\omega+\delta)\sin(\delta+\phi)}{\sin\phi'\sin\omega'}}$$

と置く　　　　・・・(13)

$$K_a=\frac{\sin^2\omega'}{\sin^2\omega}(\cot\omega'-x_a)\frac{1}{Z}(\cos\phi'-x_a\sin\phi')$$

$$=\frac{\sin^2\omega'}{\sin^2\omega}\frac{\cos\phi'}{Z}(\cot\omega'-x_a)(1-x_a\tan\phi')\quad\cdot\cdot\cdot\ \mathbf{(14)}$$

(13) $\rightarrow x_a = \dfrac{Z+\cos\mu}{\sin\mu}$ ・・・(15)

(14) $\rightarrow K_a = \dfrac{\sin^2\omega'}{\sin^2\omega}\dfrac{\cos\phi'}{Z}\left(\cot\omega' - x_a\cot\omega'\tan\phi' - x_a + x_a{}^2\tan\phi'\right)$

$$= \frac{\sin^2\omega'}{\sin^2\omega}\frac{\cos\phi'}{Z}\left(\cot\omega' - \frac{Z+\cos\mu}{\sin\mu}\cot\omega'\tan\phi' - \frac{Z+\cos\mu}{\sin\mu} + \frac{(Z+\cos\mu)^2}{\sin^2\mu}\tan\phi'\right)$$

$$= \frac{\sin^2\omega'}{\sin^2\omega}\frac{\cos\phi'}{Z\sin^2\mu}\left(\sin^2\mu\cot\omega' - (Z+\cos\mu)\sin\mu\cot\omega'\tan\phi' - (Z+\cos\mu)\sin\mu + (Z+\cos\mu)^2\tan\phi'\right)$$ ・・・

(16)

(16) のカッコ内を A とおくと

$$A = \sin^2\mu\cot\omega' - (Z+\cos\mu)\sin\mu\cot\omega'\tan\phi' - (Z+\cos\mu)\sin\mu + (Z+\cos\mu)^2\tan\phi'$$

$$= \tan\phi'\left(Z^2 + 2\cdot Z\cos\mu + \cos^2\mu\right) + \tan\phi'\cot\omega'\left(-Z\sin\mu - \sin\mu\cos\mu\right) - Z\sin\mu - \sin\mu\cos\mu + \sin^2\mu\cot\omega'$$

$$= Z^2\tan\phi' + 2\cdot Z\cos\mu\tan\phi' \underbrace{+\cos^2\mu\tan\phi'}_{A_1} - Z\sin\mu\tan\phi'\cot\omega' \underbrace{-\sin\mu\cos\mu\tan\phi'\cot\omega'}_{A_1} - Z\sin\mu \underbrace{-\sin\mu\cos\mu + \sin^2\mu\cot\omega'}_{A_2}$$

・・・(17)

$$A_1 = \cos^2\mu\tan\phi' - \sin\mu\cos\mu\tan\phi'\cot\omega'$$

$$= \cos\mu\tan\phi'(\cos\mu - \sin\mu\cot\omega')$$

$$= \cos\mu\tan\phi'\frac{\sin\omega'\cos\mu - \cos\omega'\sin\mu}{\sin\omega'}$$

$$= \cos\mu\tan\phi'\frac{\sin(\omega'-\mu)}{\sin\omega'}$$

$$= -\cos\mu\tan\phi'\frac{\sin(\delta+\phi)}{\sin\omega'}$$ ・・・(18)

$$A_2 = -\sin\mu\cos\mu + \sin^2\mu\cot\omega'$$

$$= -\sin\mu(\cos\mu - \sin\mu\cot\omega')$$

$$= -\sin\mu\frac{\sin\omega'\cos\mu - \cos\omega'\sin\mu}{\sin\omega'}$$

$$= -\sin\mu\frac{\sin(\omega'-\mu)}{\sin\omega'}$$

$$= \sin\mu\frac{\sin(\delta+\phi)}{\sin\omega'}$$ ・・・(19)

（18）、（19）より

$$A_1+A_2=-\cos\mu\tan\phi'\frac{\sin(\delta+\phi)}{\sin\omega'}+\sin\mu\frac{\sin(\delta+\phi)}{\sin\omega'}$$

$$=-\frac{\sin(\delta+\phi)}{\sin\omega'}(\cos\mu\tan\phi'-\sin\mu)$$

$$=-\frac{\sin(\delta+\phi)}{\sin\omega'}\cdot\frac{\cos\mu\sin\phi'-\sin\mu\cos\phi'}{\cos\phi'}$$

$$=-\frac{\sin(\delta+\phi)}{\sin\omega'}\cdot\frac{\sin(\phi'-\mu)}{\cos\phi'}$$

$$=\frac{\sin(\delta+\phi)\sin(\omega+\delta)}{\sin\omega'\cos\phi'} \qquad \cdot\cdot\cdot(20)$$

$$(20)=A_1+A_2=\frac{\sin(\delta+\phi)\sin(\omega+\delta)}{\sin\omega'\sin\phi'}\cdot\frac{\sin\phi'}{\cos\phi'}=Z^2\tan\phi' \qquad \cdot\cdot\cdot(21)$$

←（13）より

以上より、（16）を整理すると

$$K_a=\frac{\sin^2\omega'}{\sin^2\omega}\frac{\cos\phi'}{Z\sin^2\mu}\left(2\cdot Z^2\tan\phi'+2\cdot Z\cos\mu\tan\phi'-Z\sin\mu\tan\phi'\cot\omega'-Z\sin\mu\right)$$

$$K_a=\frac{\sin^2\omega'}{\sin^2\omega}\frac{\cos\phi'}{\sin^2\mu}\left(2\cdot Z\tan\phi'+2\cos\mu\tan\phi'-\sin\mu\tan\phi'\cot\omega'-\sin\mu\right) \qquad \cdot\cdot\cdot(22)$$

（22）のカッコ内をBと置くと

$$B=2\cdot Z\tan\phi'+2\cos\mu\tan\phi'-\sin\mu\tan\phi'\cot\omega'-\sin\mu$$

$$=2\cdot Z\tan\phi'+\underbrace{(\cos\mu\tan\phi'-\sin\mu\tan\phi'\cot\omega')}_{B_1}+\underbrace{(\cos\mu\tan\phi'-\sin\mu)}_{B_2} \qquad \cdot\cdot\cdot(23)$$

$$B_1=\cos\mu\tan\phi'-\sin\mu\tan\phi'\cot\omega'$$

$$=\tan\phi'(\cos\mu-\sin\mu\cot\omega')$$

$$=\tan\phi'\frac{\cos\mu\sin\omega'-\sin\mu\cos\omega'}{\sin\omega'}$$

$$=\tan\phi'\frac{\sin(\omega'-\mu)}{\sin\omega'}$$

$$=-\tan\phi'\frac{\sin(\delta+\phi)}{\sin\omega'} \qquad \cdot\cdot\cdot(24)$$

$$B_2 = \cos\mu\tan\phi' - \sin\mu$$

$$= \frac{\cos\mu\sin\phi' - \cos\phi'\sin\mu}{\cos\phi'}$$

$$= \frac{\sin(\phi' - \mu)}{\cos\phi'}$$

$$= -\frac{\sin(\omega + \delta)}{\cos\phi'} \qquad \cdot\cdot\cdot\ (\mathbf{25})$$

（24）、（25）→（22）

$$K_a = \frac{\sin^2\omega'}{\sin^2\omega}\frac{\cos\phi'}{\sin^2\mu}\left(2\cdot Z\tan\phi' - \tan\phi'\frac{\sin(\delta+\phi)}{\sin\omega'} - \frac{\sin(\omega+\delta)}{\cos\phi'}\right)$$

$$= -\frac{\sin^2\omega'}{\sin^2\omega}\frac{\cos\phi'}{\sin^2\mu}\left(\frac{\tan\phi'}{\sin\omega'}\sin(\delta+\phi) + 2\tan\phi'\sqrt{\frac{\sin(\omega+\delta)\sin(\delta+\phi)}{\sin\phi'\sin\omega'}} + \frac{\sin(\omega+\delta)}{\cos\phi'}\right)$$

$$= -\frac{\sin^2\omega'}{\sin^2\omega}\frac{\cos\phi'}{\sin^2\mu}\left(\sqrt{\frac{\tan\phi'}{\sin\omega'}\sin(\delta+\phi)} + \sqrt{\frac{\sin(\omega+\delta)}{\cos\phi'}}\right)^2$$

$$= -\frac{\sin^2\omega'}{\sin^2\omega}\frac{\cos\phi'}{\sin^2\mu}\frac{\left(\frac{\tan\phi'}{\sin\omega'}\sin(\delta+\phi) - \frac{\sin(\omega+\delta)}{\cos\phi'}\right)^2}{\left(\sqrt{\frac{\tan\phi'}{\sin\omega'}\sin(\delta+\phi)} - \sqrt{\frac{\sin(\omega+\delta)}{\cos\phi'}}\right)^2} \longleftarrow$$

$$\left(\sqrt{x}+\sqrt{y}\right)^2$$
$$= x + 2\sqrt{x}\sqrt{y} + y$$
$$= \frac{\left(x + y;2\sqrt{x}\sqrt{y}\right)\left(x + y - 2\sqrt{x}\sqrt{y}\right)}{\left(x - 2\sqrt{x}\sqrt{y} + y\right)}$$
$$= \frac{(x+y)^2 - \left(2\sqrt{x}\sqrt{y}\right)^2}{\left(x - 2\sqrt{x}\sqrt{y} + y\right)}$$
$$= \frac{(x-y)^2}{\left(\sqrt{x}-\sqrt{y}\right)^2}$$

$$= -\frac{\sin^2\omega'}{\sin^2\omega}\frac{\cos\phi'}{\sin^2\mu}\frac{1}{\left(\sin^2\omega'\cos^2\phi'\right)}\frac{\left(\sin\phi'^2\sin^2(\delta+\phi) - 2\cdot\sin\omega'\sin\phi'\sin(\delta+\phi)\sin(\omega+\delta) + \sin^2\omega'\sin^2(\omega+\delta)\right)}{\left(\frac{\tan\phi'}{\sin\omega'}\sin(\delta+\phi) - 2\cdot\sqrt{\frac{\sin(\delta+\phi)\sin(\omega+\delta)}{\sin\omega'\cos\phi'}}\tan\phi' + \frac{\sin(\omega+\delta)}{\cos\phi'}\right)^2}$$

$$= -\frac{\left(\sin\phi'\sin(\delta+\phi) - \sin\omega'\sin(\omega+\delta)\right)^2}{\sin^2\omega\cdot\sin^2\mu\left(\sqrt{\sin(\omega+\delta)} - \sqrt{\frac{\sin\phi'}{\sin\omega'}}\sin(\delta+\phi)\right)^2} \qquad \cdot\cdot\cdot\ (\mathbf{26})$$

ここで（26）の分子を以下のように置く

$$[C]^2 = \left(\sin\phi'\sin(\delta+\phi) - \sin\omega'\sin(\omega+\delta)\right)^2$$

$$C=\sin\phi'\sin(\delta+\phi)-\sin\omega'\sin(\omega+\delta)$$

$$=\frac{1}{2}\left(-\cos(\delta+\phi+\phi')+\cos(\delta+\phi-\phi')+\cos(\omega+\delta+\omega')-\cos(\omega+\delta-\omega')\right)$$

$$=\frac{1}{2}\left(-\cos(\delta+2\phi-\beta)+\cos(\delta+\beta)+\cos(2\omega+\delta-\beta)-\cos(\delta+\beta)\right)$$

$$=\frac{1}{2}\left(\cos(2\omega+\delta-\beta)-\cos(\delta+2\phi-\beta)\right)$$

$$=-\left(\sin\frac{(2\omega+\delta-\beta)+(\delta+2\phi-\beta)}{2}\sin\frac{(2\omega+\delta-\beta)-(\delta+2\phi-\beta)}{2}\right)$$

$$=-\sin(\omega+\delta+\phi-\beta)\cdot\sin(\omega-\phi)$$

$$=-\sin\mu\cdot\sin(\omega-\phi) \qquad \cdot\cdot\cdot\ \mathbf{(27)}$$

（27）→（26）

$$K_a=\frac{(\sin\mu\cdot\sin(\omega-\phi))^2}{\sin^2\omega\cdot\sin^2\mu\left(\sqrt{\sin(\omega+\delta)}-\sqrt{\frac{\sin\phi'}{\sin\omega'}\sin(\delta+\phi)}\right)^2}$$

$$=\frac{\sin^2(\omega-\phi)}{\sin^2\omega\cdot\left(\sqrt{\sin(\omega+\delta)}-\sqrt{\frac{\sin(\phi-\beta)\sin(\delta+\phi)}{\sin\omega'}}\right)^2}$$

$$=\left[\frac{\sin(\omega-\phi)}{\sin\omega\cdot\left(\sqrt{\sin(\omega+\delta)}-\sqrt{\frac{\sin(\phi-\beta)\sin(\delta+\phi)}{\sin\omega'}}\right)}\right]^2$$

以上より主働土圧について Coulomb 土圧が導かれた。

受動土圧はδ、φの符号が逆の場合であり、その導入は同様であり省略する。

以上