The quantum mechanics canonically associated to free probability

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Abstract

Probably the most relevant discovery in the past 20 years in quantum probability has been the explanation of the old mystery about the origins of quantization.

Now we know that every probability measure with all moments is canonically associated with its own quantum theory in the sense that it canonically defines 3 families of operators, called creators annihilators and preservators (CAP), and a family of commutation relations satisfied by these operators.

The usual Heisenberg relations characterize Gaussian measures.

This is true in finite or infinite dimensions, so that quantum field theory is included.

In the past 100 years physicists have learnt to model many important natural phenomena using the only quantum mechanics known up to now: Heisenberg (or Gaussian) QM.

The new approach to quantization opens up new avenues for the modeling of natural phenomena in fact, since canonical objects in mathematics eventually turn out to be related to interesting physical phenomena, the program to study the QM associated to the simplest and most natural classes of probability measures, greatly empowers the possibility to build models that, looked from the point of view of usual QM appear to be highly non–linear.

According to the recently introduced information complexity index for probability measures, the simplest probability measure with infinite support is the semicircle law which is well known to be the 'Gaussian' for quantum probability. Thus the QM of the semicircle law is the quantum mechanics canonically associated to free probability.

This QM introduces some unexpected features.
For example we will see that in it the canonically conjugate momentum, associated to a classical the semicircle random variable, is $-i$ times the μ–Hilbert transform, Where μ is the semicircle distribution.