Coulomb 土圧の導入

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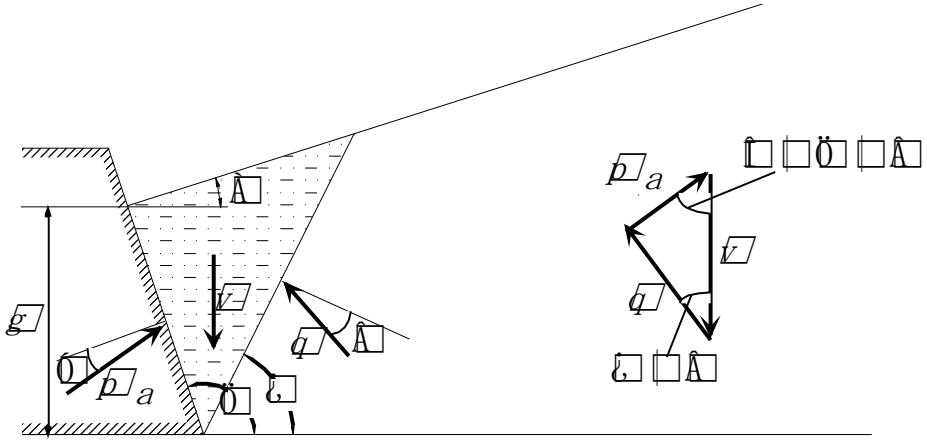
Coulomb 土圧は次式で与えられる。

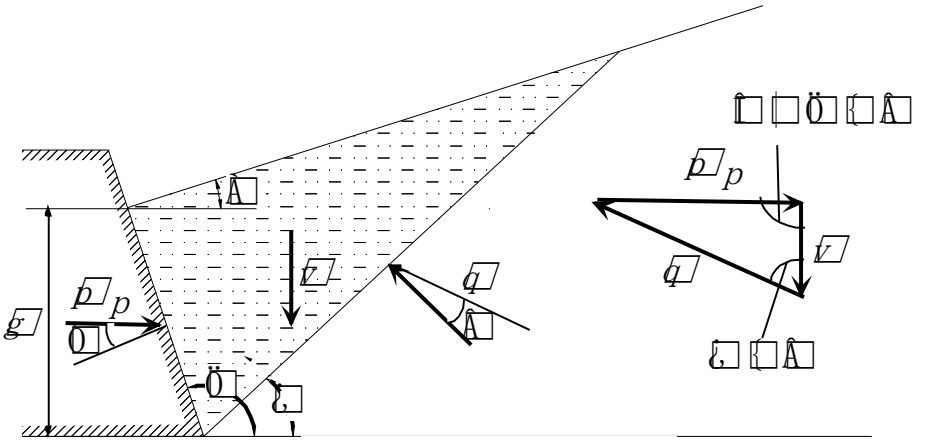
$$\begin{pmatrix} Q_a \\ Q_p \end{pmatrix} = \frac{1}{2}\gamma_t H^2 \begin{pmatrix} K_a \\ K_p \end{pmatrix}$$

$$\begin{pmatrix} K_a \\ K_p \end{pmatrix} = \left[\frac{\sin(\omega \mp \phi)}{\sin\omega\left\{\sqrt{\sin(\omega \pm \delta)} \pm \sqrt{\dfrac{\sin(\phi+\delta)\sin(\phi \mp \beta)}{\sin(\omega-\beta)}}\right\}} \right]$$

この土圧係数 K_a,K_p を以下誘導する。



(a)主働土圧



(b)受動土圧

Fig. 1

破壊領域の重量Wの計算（Fig.2）

$$W = \frac{1}{2}\gamma_t \cdot \overline{\mathrm{AB}} \cdot \overline{\mathrm{BC}} \cdot \sin(\omega - \alpha) \quad \cdots (1)$$

$$\overline{\mathrm{AB}} = \frac{H}{\sin\omega} \quad \cdots (2)$$

$$\frac{\overline{\mathrm{AB}}}{\sin(\angle \mathrm{C})} = \frac{\overline{\mathrm{BC}}}{\sin(\angle \mathrm{A})} \quad \cdots (正弦定理)$$

$$\overline{\mathrm{BC}} = \frac{H}{\sin\omega} \cdot \frac{\sin(\pi - \omega + \beta)}{\sin(\alpha - \beta)}$$

$$= \frac{H}{\sin\omega} \cdot \frac{\sin(\omega - \beta)}{\sin(\alpha - \beta)} \quad \cdots (3)$$

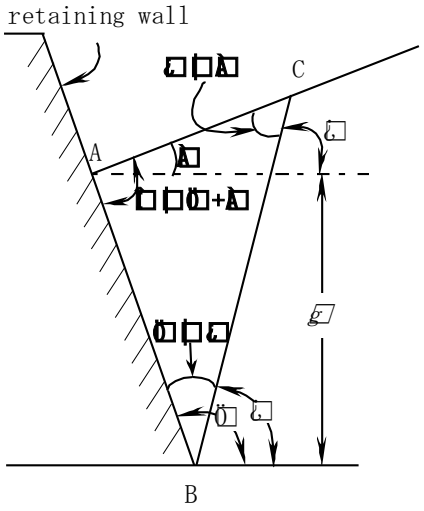


Fig. 2

（２）、（３）を（１）に代入

$$W = \frac{1}{2}\gamma_t H^2 \frac{\sin(\omega - \beta)\sin(\omega - \alpha)}{\sin^2\omega \sin(\alpha - \beta)}$$

$$= \frac{1}{2}\gamma_t H^2 \frac{\sin(\omega - \beta)\sin\{(\omega - \beta) - (\alpha - \beta)\}}{\sin^2\omega \sin(\alpha - \beta)}$$

$$= \frac{1}{2}\gamma_t H^2 \frac{\sin(\omega - \beta)\{\sin(\omega - \beta)\cos(\alpha - \beta) - \cos(\omega - \beta)\sin(\alpha - \beta)\}}{\sin^2\omega \sin(\alpha - \beta)}$$

$$= \frac{1}{2}\gamma_t H^2 \frac{\sin^2(\omega - \beta)}{\sin^2\omega}\{\cot(\alpha - \beta) - \cot(\omega - \beta)\} \quad \cdots (4)$$

力の釣り合い（Fig.3）

$$\frac{W}{\sin(\omega + \delta + \phi - \alpha)} = \frac{Q}{\sin(\alpha - \phi)} \quad \cdots (正弦定理)$$

$$Q = W \frac{\sin(\alpha - \phi)}{\sin(\omega + \delta + \phi - \alpha)}$$

$$= W \frac{\sin\{(\alpha - \beta) - (\phi - \beta)\}}{\sin\{(\omega + \delta + \phi - \beta) - (\alpha - \beta)\}}$$

$$= W \frac{\sin(\alpha - \beta)\cos(\phi - \beta) - \cos(\alpha - \beta)\sin(\phi - \beta)}{\sin(\omega + \delta + \phi - \beta)\cos(\alpha - \beta) - \cos(\omega + \delta + \phi - \beta)\sin(\alpha - \beta)}$$

$$= W \frac{\cos(\phi - \beta) - \sin(\phi - \beta)\cot(\alpha - \beta)}{\sin(\omega + \delta + \phi - \beta)\cot(\alpha - \beta) - \cos(\omega + \delta + \phi - \beta)} \quad \cdots (5)$$

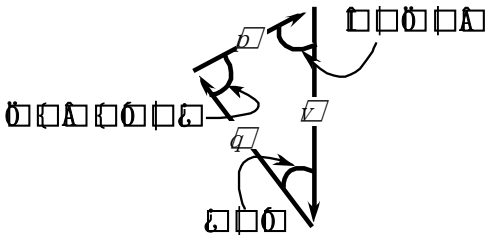


Fig. 3

（４）を（５）に代入

$$Q=\frac{1}{2}\gamma_t\frac{\sin^2(\omega-\beta)}{\sin^2\omega}\{\cot(\alpha-\beta)-\cot(\omega-\beta)\}\frac{\cos(\phi-\beta)-\sin(\phi-\beta)\cot(\alpha-\beta)}{\sin(\omega+\delta+\phi-\beta)\cot(\alpha-\beta)-\cos(\omega+\delta+\phi-\beta)}\quad\cdots(6)$$

$$\left\{\begin{array}{c}x=\cot(\alpha-\beta)\\ \mu=\omega+\delta+\phi-\beta\\ \phi'=\phi-\beta\\ \omega'=\omega-\beta\end{array}\right\}\cdots(7)\qquad\left\{\begin{array}{l}\mu-\phi'=\omega+\delta\\ \mu-\omega'=\delta+\phi\end{array}\right\}\cdots(7)'$$

（７）より

$$Q=\frac{1}{2}\gamma_t\frac{\sin^2\omega'}{\sin^2\omega}\{x-\cot\omega'\}\frac{\cos\phi'-x\sin\phi'}{x\sin\mu-\cos\mu}\quad\cdots(8)$$

（８）より土圧合力 Q は $x=\cot(\alpha-\beta)$ の関数である。

主働土圧 Q_a は土圧合力 Q が最小となる場合であるので、$\partial Q/\partial x=0$ を満たす x_a を決め、そのときの土圧を求めればよい。

$$\frac{\partial Q}{\partial x}=\frac{\partial}{\partial x}\left[\frac{1}{2}\gamma_t\frac{\sin^2\omega'}{\sin^2\omega}\{x-\cot\omega'\}\frac{\cos\phi'-x\sin\phi'}{x\sin\mu-\cos\mu}\right]=0$$

$$\Leftrightarrow\frac{\partial}{\partial x}\left[(x-\cot\omega')\frac{\cos\phi'-x\sin\phi'}{x\sin\mu-\cos\mu}\right]=0$$

$$\Leftrightarrow\frac{\cos\phi'-x\sin\phi'}{x\sin\mu-\cos\mu}+(x-\cot\omega')\frac{\partial}{\partial x}\left[\frac{\cos\phi'-x\sin\phi'}{x\sin\mu-\cos\mu}\right]=0$$

$$\Leftrightarrow\frac{\cos\phi'-x\sin\phi'}{x\sin\mu-\cos\mu}+(x-\cot\omega')\left\{\frac{(-\sin\phi')(x\sin\mu-\cos\mu)-\sin\mu(\cos\phi'-x\sin\phi')}{(x\sin\mu-\cos\mu)^2}\right\}=0$$

$$\Leftrightarrow\frac{\cos\phi'-x\sin\phi'}{x\sin\mu-\cos\mu}+\frac{(x-\cot\omega')(-\sin\phi')}{x\sin\mu-\cos\mu}+\frac{-\sin\mu(x-\cot\omega')(\cos\phi'-x\sin\phi')}{(x\sin\mu-\cos\mu)^2}=0$$

$$\Leftrightarrow(x\sin\mu-\cos\mu)(\cos\phi'-x\sin\phi')+(x-\cot\omega')(-\sin\phi')(x\sin\mu-\cos\mu)+(-\sin\mu)(x-\cot\omega')(\cos\phi'-x\sin\phi')=0$$

展開して整理

$$(-\sin\mu\sin\phi')x^2+2(\cos\mu\sin\phi')x-(\cos\mu\cos\phi'+\cot\omega'\cos\mu\sin\phi'-\cot\omega'\sin\mu\cos\phi')=0\quad\cdots(9)$$

（９）の正の解

$$x_a=\frac{1}{(-\sin\mu\sin\phi')}\left\{-\cos\mu\sin\phi'+\sqrt{(\cos\mu\sin\phi')^2+(-\sin\mu\sin\phi')(\cos\mu\cos\phi'+\cos\omega\cos\mu\sin\phi'-\cot\omega'\sin\mu\cos\phi')}\right\}$$

$$\cdots(10)$$

（10）の$\sqrt{\ }$内

$$(\cos\mu\sin\phi')^2+(-\sin\mu\sin\phi')(\cos\mu\cos\phi'+\cos\omega\cos\mu\sin\phi'-\cot\omega'\sin\mu\cos\phi')$$
$$=\cos^2\mu\sin^2\phi'-\sin\mu\sin\phi'\cos\mu\cos\phi'-\sin\mu\sin^2\phi'\cos\mu\cot\omega'+\sin\mu\sin\phi'\sin\mu\cos\phi'\cot\omega'$$
$$=\frac{\sin\phi'}{\sin\omega'}\left(\cos^2\mu\sin\phi'\sin\omega'-\sin\mu\cos\mu\cos\phi'\sin\omega'-\sin\mu\sin\phi'\cos\mu\cos\omega'+\sin\mu\sin\mu\cos\phi'\cos\omega'\right)$$
$$=\frac{\sin\phi'}{\sin\omega'}(\cos\mu\sin\phi'-\sin\mu\cos')(\cos\mu\sin\omega'-\sin\mu\cos\omega')$$
$$=\frac{\sin\phi'}{\sin\omega'}\sin(\phi'-\mu)\sin(\omega'-\mu)$$
$$=\frac{\sin\phi'}{\sin\omega'}\sin(\omega+\delta)\sin(\delta+\phi)\qquad\leftarrow（7）'より$$

（10）に戻って

$$x_a=\frac{1}{(-\sin\mu\sin\phi')}\left\{-\cos\mu\sin\phi'+\sqrt{\frac{\sin\phi'}{\sin\omega'}\sin(\omega+\delta)\sin(\delta+\phi)}\right\}$$

$$x_a=\cot\mu-\frac{1}{\sin\mu}\sqrt{\frac{\sin(\omega+\delta)\sin(\delta+\phi)}{\sin\phi'\sin\omega'}}\quad\cdots\textbf{(11)}$$

$x=x_a$のとき$Q=Q_a$　　（8）より

$$\left\{\begin{array}{l}Q_a=\dfrac{1}{2}\gamma_t K_a\\ K_a=\dfrac{\sin^2\omega'}{\sin^2\omega}\{x_a-\cot\omega'\}\dfrac{\cos\phi'-x_a\sin\phi'}{x_a\sin\mu-\cos\mu}\end{array}\right\}\quad\cdots\textbf{(12)}$$

$x_a\sin\mu-\cos\mu$ ← Z

$$Z=x_a\sin\mu-\cos\mu=\left(\cot\mu-\frac{1}{\sin\mu}\sqrt{\frac{\sin(\omega+\delta)\sin(\delta+\phi)}{\sin\phi'\sin\omega'}}\right)\sin\mu-\cos\mu=-\sqrt{\frac{\sin(\omega+\delta)\sin(\delta+\phi)}{\sin\phi'\sin\omega'}}$$

と置く　　**・・・(13)**

$$K_a=\frac{\sin^2\omega'}{\sin^2\omega}(\cot\omega'-x_a)\frac{1}{Z}(\cos\phi'-x_a\sin\phi')$$
$$=\frac{\sin^2\omega'}{\sin^2\omega}\frac{\cos\phi'}{Z}(\cot\omega'-x_a)(1-x_a\tan\phi')\qquad\cdots\textbf{(14)}$$

（13）$\rightarrow x_a = \dfrac{Z+\cos\mu}{\sin\mu}$ ・・・**(15)**

（14）$\rightarrow K_a = \dfrac{\sin^2\omega'}{\sin^2\omega}\dfrac{\cos\phi'}{Z}\left(\cot\omega' - x_a\cot\omega'\tan\phi' - x_a + {x_a}^2\tan\phi'\right)$

$$= \frac{\sin^2\omega'}{\sin^2\omega}\frac{\cos\phi'}{Z}\left(\cot\omega' - \frac{Z+\cos\mu}{\sin\mu}\cot\omega'\tan\phi' - \frac{Z+\cos\mu}{\sin\mu} + \frac{(Z+\cos\mu)^2}{\sin^2\mu}\tan\phi'\right)$$

$$= \frac{\sin^2\omega'}{\sin^2\omega}\frac{\cos\phi'}{Z\sin^2\mu}\left(\sin^2\mu\cot\omega' - (Z+\cos\mu)\sin\mu\cot\omega'\tan\phi' - (Z+\cos\mu)\sin\mu + (Z+\cos\mu)^2\tan\phi'\right)$$ ・・・**(16)**

（16）のカッコ内を A とおくと

$$A = \sin^2\mu\cot\omega' - (Z+\cos\mu)\sin\mu\cot\omega'\tan\phi' - (Z+\cos\mu)\sin\mu + (Z+\cos\mu)^2\tan\phi'$$

$$= \tan\phi'\left(Z^2 + 2\cdot Z\cos\mu + \cos^2\mu\right) + \tan\phi'\cot\omega'\left(-Z\sin\mu - \sin\mu\cos\mu\right) - Z\sin\mu - \sin\mu\cos\mu + \sin^2\mu\cot\omega'$$

$$= Z^2\tan\phi' + 2\cdot Z\cos\mu\tan\phi' + \cos^2\mu\tan\phi' - Z\sin\mu\tan\phi'\cot\omega' - \sin\mu\cos\mu\tan\phi'\cot\omega' - Z\sin\mu - \sin\mu\cos\mu + \sin^2\mu\cot\omega'$$

A_1 A_1 A_2

・・・**(17)**

$$A_1 = \cos^2\mu\tan\phi' - \sin\mu\cos\mu\tan\phi'\cot\omega'$$

$$= \cos\mu\tan\phi'\left(\cos\mu - \sin\mu\cot\omega'\right)$$

$$= \cos\mu\tan\phi'\frac{\sin\omega'\cos\mu - \cos\omega'\sin\mu}{\sin\omega'}$$

$$= \cos\mu\tan\phi'\frac{\sin(\omega'-\mu)}{\sin\omega'}$$

$$= -\cos\mu\tan\phi'\frac{\sin(\delta+\phi)}{\sin\omega'}$$ ・・・**(18)**

$$A_2 = -\sin\mu\cos\mu + \sin^2\mu\cot\omega'$$

$$= -\sin\mu\left(\cos\mu - \sin\mu\cot\omega'\right)$$

$$= -\sin\mu\frac{\sin\omega'\cos\mu - \cos\omega'\sin\mu}{\sin\omega'}$$

$$= -\sin\mu\frac{\sin(\omega'-\mu)}{\sin\omega'}$$

$$= \sin\mu\frac{\sin(\delta+\phi)}{\sin\omega'}$$ ・・・**(19)**

（18）、（19）より

$$A_1 + A_2 = -\cos\mu\tan\phi'\frac{\sin(\delta+\phi)}{\sin\omega'} + \sin\mu\frac{\sin(\delta+\phi)}{\sin\omega'}$$

$$= -\frac{\sin(\delta+\phi)}{\sin\omega'}(\cos\mu\tan\phi' - \sin\mu)$$

$$= -\frac{\sin(\delta+\phi)}{\sin\omega'}\cdot\frac{\cos\mu\sin\phi' - \sin\mu\cos\phi'}{\cos\phi'}$$

$$= -\frac{\sin(\delta+\phi)}{\sin\omega'}\cdot\frac{\sin(\phi'-\mu)}{\cos\phi'}$$

$$= \frac{\sin(\delta+\phi)\sin(\omega+\delta)}{\sin\omega'\cos\phi'} \quad \cdots (20)$$

$$(20) = A_1 + A_2 = \frac{\sin(\delta+\phi)\sin(\omega+\delta)}{\sin\omega'\sin\phi'}\cdot\frac{\sin\phi'}{\cos\phi'} = Z^2\tan\phi' \quad \cdots (21) \quad \leftarrow (13)\text{ より}$$

以上より、（16）を整理すると

$$K_a = \frac{\sin^2\omega'}{\sin^2\omega}\frac{\cos\phi'}{Z\sin^2\mu}\left(2\cdot Z^2\tan\phi' + 2\cdot Z\cos\mu\tan\phi' - Z\sin\mu\tan\phi'\cot\omega' - Z\sin\mu\right)$$

$$K_a = \frac{\sin^2\omega'}{\sin^2\omega}\frac{\cos\phi'}{\sin^2\mu}\left(2\cdot Z\tan\phi' + 2\cos\mu\tan\phi' - \sin\mu\tan\phi'\cot\omega' - \sin\mu\right) \quad \cdots (22)$$

（22）のカッコ内をBと置くと

$$B = 2\cdot Z\tan\phi' + 2\cos\mu\tan\phi' - \sin\mu\tan\phi'\cot\omega' - \sin\mu$$

$$= 2\cdot Z\tan\phi' + \underbrace{(\cos\mu\tan\phi' - \sin\mu\tan\phi'\cot\omega')}_{B_1} + \underbrace{(\cos\mu\tan\phi' - \sin\mu)}_{B_2} \quad \cdots (23)$$

$$B_1 = \cos\mu\tan\phi' - \sin\mu\tan\phi'\cot\omega'$$

$$= \tan\phi'(\cos\mu - \sin\mu\cot\omega')$$

$$= \tan\phi'\frac{\cos\mu\sin\omega' - \sin\mu\cos\omega'}{\sin\omega'}$$

$$= \tan\phi'\frac{\sin(\omega'-\mu)}{\sin\omega'}$$

$$= -\tan\phi'\frac{\sin(\delta+\phi)}{\sin\omega'} \quad \cdots (24)$$

$$B_2 = \cos\mu\tan\phi' - \sin\mu$$

$$= \frac{\cos\mu\sin\phi' - \cos\phi'\sin\mu}{\cos\phi'}$$

$$= \frac{\sin(\phi' - \mu)}{\cos\phi'}$$

$$= -\frac{\sin(\omega + \delta)}{\cos\phi'} \qquad \cdots (25)$$

（24）、（25）→（22）

$$K_a = \frac{\sin^2\omega'}{\sin^2\omega}\frac{\cos\phi'}{\sin^2\mu}\left(2\cdot Z\tan\phi' - \tan\phi'\frac{\sin(\delta+\phi)}{\sin\omega'} - \frac{\sin(\omega+\delta)}{\cos\phi'}\right)$$

$$= -\frac{\sin^2\omega'}{\sin^2\omega}\frac{\cos\phi'}{\sin^2\mu}\left(\frac{\tan\phi'}{\sin\omega'}\sin(\delta+\phi) + 2\tan\phi'\sqrt{\frac{\sin(\omega+\delta)\sin(\delta+\phi)}{\sin\phi'\sin\omega'}} + \frac{\sin(\omega+\delta)}{\cos\phi'}\right)$$

$$= -\frac{\sin^2\omega'}{\sin^2\omega}\frac{\cos\phi'}{\sin^2\mu}\left(\sqrt{\frac{\tan\phi'}{\sin\omega'}\sin(\delta+\phi)} + \sqrt{\frac{\sin(\omega+\delta)}{\cos\phi'}}\right)^2$$

$$= -\frac{\sin^2\omega'}{\sin^2\omega}\frac{\cos\phi'}{\sin^2\mu}\frac{\left(\frac{\tan\phi'}{\sin\omega'}\sin(\delta+\phi) - \frac{\sin(\omega+\delta)}{\cos\phi'}\right)^2}{\left(\sqrt{\frac{\tan\phi'}{\sin\omega'}\sin(\delta+\phi)} - \sqrt{\frac{\sin(\omega+\delta)}{\cos\phi'}}\right)^2}$$

$$\left(\sqrt{x}+\sqrt{y}\right)^2$$
$$= x + 2\sqrt{x}\sqrt{y} + y$$
$$= \frac{\left(x+y;2\sqrt{x}\sqrt{y}\right)\left(x+y-2\sqrt{x}\sqrt{y}\right)}{\left(x-2\sqrt{x}\sqrt{y}+y\right)}$$
$$= \frac{(x+y)^2 - \left(2\sqrt{x}\sqrt{y}\right)^2}{\left(x-2\sqrt{x}\sqrt{y}+y\right)}$$
$$= \frac{(x-y)^2}{\left(\sqrt{x}-\sqrt{y}\right)^2}$$

$$= -\frac{\sin^2\omega'}{\sin^2\omega}\frac{\cos\phi'}{\sin^2\mu}\frac{1}{\left(\sin^2\omega'\cos^2\phi'\right)}\frac{\left(\sin\phi'^2\sin^2(\delta+\phi) - 2\cdot\sin\omega'\sin\phi'\sin(\delta+\phi)\sin(\omega+\delta) + \sin^2\omega'\sin^2(\omega+\delta)\right)}{\left(\frac{\tan\phi'}{\sin\omega'}\sin(\delta+\phi) - 2\cdot\sqrt{\frac{\sin(\delta+\phi)\sin(\omega+\delta)}{\sin\omega'\cos\phi'}\tan\phi'} + \frac{\sin(\omega+\delta)}{\cos\phi'}\right)^2}$$

$$= -\frac{\left(\sin\phi'\sin(\delta+\phi) - \sin\omega'\sin(\omega+\delta)\right)^2}{\sin^2\omega\cdot\sin^2\mu\left(\sqrt{\sin(\omega+\delta)} - \sqrt{\frac{\sin\phi'}{\sin\omega'}\sin(\delta+\phi)}\right)^2} \qquad \cdots (26)$$

ここで（26）の分子を以下のように置く

$$[C]^2 = \left(\sin\phi'\sin(\delta+\phi) - \sin\omega'\sin(\omega+\delta)\right)^2$$

$$C = \sin\phi' \sin(\delta+\phi) - \sin\omega' \sin(\omega+\delta)$$

$$= \frac{1}{2}\left(-\cos(\delta+\phi+\phi') + \cos(\delta+\phi-\phi') + \cos(\omega+\delta+\omega') - \cos(\omega+\delta-\omega')\right)$$

$$= \frac{1}{2}\left(-\cos(\delta+2\phi-\beta) + \cos(\delta+\beta) + \cos(2\omega+\delta-\beta) - \cos(\delta+\beta)\right)$$

$$= \frac{1}{2}\left(\cos(2\omega+\delta-\beta) - \cos(\delta+2\phi-\beta)\right)$$

$$= -\left(\sin\frac{(2\omega+\delta-\beta)+(\delta+2\phi-\beta)}{2}\sin\frac{(2\omega+\delta-\beta)-(\delta+2\phi-\beta)}{2}\right)$$

$$= -\sin(\omega+\delta+\phi-\beta)\cdot\sin(\omega-\phi)$$

$$= -\sin\mu\cdot\sin(\omega-\phi) \qquad \cdots (27)$$

（27）→（26）

$$K_a = \frac{(\sin\mu\cdot\sin(\omega-\phi))^2}{\sin^2\omega\cdot\sin^2\mu\left(\sqrt{\sin(\omega+\delta)} - \sqrt{\frac{\sin\phi'}{\sin\omega'}\sin(\delta+\phi)}\right)^2}$$

$$= \frac{\sin^2(\omega-\phi)}{\sin^2\omega\cdot\left(\sqrt{\sin(\omega+\delta)} - \sqrt{\frac{\sin(\phi-\beta)\sin(\delta+\phi)}{\sin\omega'}}\right)^2}$$

$$= \left[\frac{\sin(\omega-\phi)}{\sin\omega\cdot\left(\sqrt{\sin(\omega+\delta)} - \sqrt{\frac{\sin(\phi-\beta)\sin(\delta+\phi)}{\sin\omega'}}\right)}\right]^2$$

以上より主働土圧について Coulomb 土圧が導かれた。

受動土圧はδ、φの符号が逆の場合であり、その導入は同様であり省略する。

以上