Deformation and Strength Characteristics of Granular Materials:
from experimental research for the last 35 years by a geotechnical engineering researcher

Tatsuoka, F.
Department of Civil Engineering
Tokyo University of Science
Shear stress, $\tau$  
Shear strain, $\gamma$  
(averaged for a specimen)  

Behaviour at small strains  
Rate effects  
Peak strength  
Strain softening  
Dilatancy  

Non-linear pre-peak stress-strain behaviour  
(Hypo) Elasticity  
Plasticity  
Viscosity  

Inherent anisotropy  
Shear banding with particle size effects  
Pressure-dependency  
Ageing effect  

Pressure-dependency
**Introduction:** Background of the research and a very brief summary of the lectures

**Elasticity:** Stress-strain behaviour at strains less than about 0.001 %

**Non-linear pre-peak stress-strain behaviour:** Mainly some effects of stress history and the shape of yield locus

**Effects of confining pressure on stress-strain behaviour:** Mainly those at relatively low and very low pressures

**Inherent anisotropy in the pre-peak stress-strain behaviour and peak strength:** experimental results only

**Shear banding:** mainly analysis related to particle size effects

**Time effects:** mainly viscous property and partly ageing effects
Typical time effects

- Creep deformation with time by sustained loading
- Stress relaxation with a fixed strain
- Increase in the strain rate
Causes for time effects on the stress-strain behaviour:
1) Delayed dissipation of excess pore water pressure;
2) Material viscosity; and
3) Ageing effect
Causes for time effects on the stress-strain behaviour:
1) Delayed dissipation of excess pore water pressure (out of scope in this lecture);
2) Material viscosity (the first topic); and
3) Ageing effect (the second topic)

Increase in the strain rate

Creep deformation with time by sustained loading

Stress relaxation with a fixed strain
References:


Time effects: mainly viscous property and partly ageing effects

1. Background
2. Definitions of ageing effect and loading rate effect
3. Some fundamentals for modelling of viscosity property
4. Isotach viscous property
5. TESRA viscous property
6. P&N viscous property
7. Non-linear three-component model
8. Effects of particle characteristics on viscous properties
9. Creep strain and cyclic loading-induced residual strain
10. Ageing effect and its modelling
Kansai International Airport
*Prediction by the classical consolidation theory (not taking into account the viscous properties of clay)

A very good prediction until the opening to service!

But.....
This trend is due to the viscosity of clay.

Prediction by the classical consolidation theory not taking into account the material viscosity.

This trend is due to the viscosity of clay.
The longest suspension bridge, but the worst ground conditions ever for long suspension bridges in Japan
Time-history of the settlement of Pier Foundation 2P on a gravel deposit for Akashi Strait Bridge

End of tower construction

The 1995 Hyogo-ken Nambu Earthquake

14th Oct., 1989

10 years
- Instantaneous settlement
- Residual settlement
Creep settlement
Settlement by seismic effects

Pier 2P of Akashi Strait Bridge

Applied pressure at the footing base, $p_{ave}$ (MPa)

Settlement, $S$ (mm)

End of tower construction

The 1995 Hygo-ken Nambu Earthquake

14th Oct., 1989

10 years

Elasped time (days)
Full-scale behaviour of pier 3P

- Over-estimation of the instantaneous settlement when based on the conventional methods.
- Accurate simulation by FEM based on \[ G_f = \rho \cdot V_s^2 \]
- Yet, the creep deformation cannot be predicted.

\[(Tatsuoka & Kohata, 1995)\]
Lower stiffness when constructed more slowly
Noticeable creep settlement during the cease of construction

\[ S^e = \text{elastic component, based on } (V_s)_{\text{field}} \]
& its pressure-dependency from laboratory stress-strain tests

\[ S^r = S^t - S^e \]
\[ S^t = \text{total settlement as measured} \]
\[ S^r = \text{Settlement rate} \]
\[ \dot{S}^r = 0.1 \text{mm/day} \]

Pier 2P of Akashi Strait Bridge

Tatsuoka et al. (1999d)
Simulation of the settlement dealing with the ground as a soil element that follows a non-linear three-component rheology model of Isotach type (explained later)

Tatsuoka et al. (1999d)
Lower stiffness when constructed more slowly
Noticeable creep settlement during the cease of construction

Tatsuoka et al. (1999d)
Simulation of the settlement dealing with the ground as a soil element that follows a non-linear three-component rheology model of Isotach type (explained later)

![Graph showing the relationship between average contact pressure and irreversible settlement. The graph includes observations and simulations, with a note on Displacement control.]

Tatsuoka et al. (1999d)
Settlements at a UK nuclear power station
First described by Dun (1973)

(Jardine, 2005)
Geology for a UK power station

(Jardine, 2005)
Long term settlements at four locations: large creep deformation of dense sand

(Jardine, 2005)
Long term settlements for four locations
Semi-logarithmicmic trends

Does sand exhibit significant creep deformation?

(Jardine, 2005)
Toyoura sand (mostly quartz, $D_{50} = 0.16$ mm)
Typical test result showing the viscosity of sand

Axial strain measured using LDTs (%)

Deviator stress, \( q = \sigma_1 - \sigma_3 \) (kPa)

Axial strain rate = 1.077 %/min.

Creep loading for one day (C)
Stress relaxation for one day (R)

Air-dried
Toyoura sand
(e= 0.658)
Drained TC
\( \sigma_h = 400 \text{ kPa} \)
Proximity transducer
Local deformation transducer
Bedding error
Local axial strain
Pressure cell
Axial strain including B.E.
Triaxial testing system for small specimens locally measuring axial strains developed at the IIS, the University of Tokyo
How to predict the time-dependent behaviour for general loading histories?

e.g., creep deformation at constant footing load

- Prediction of
  1) Behaviours at different construction speeds; 
     \((a\rightarrow b)\) versus \((a\rightarrow c)\)
  2) Rates of creep and stress relaxation after different construction speeds; 
     \((b\rightarrow d)\) versus \((c\rightarrow e)\); and 
     \((b\rightarrow d')\) versus \((c\rightarrow e')\)
Time-dependent behaviour under unloaded conditions

3) Behaviour after creep; (e→f→g)

4) Creep and stress relaxation after unloading; (h→i) and (h→j)
Several reasons why laboratory study on the loading rate of geomaterial is difficult.

It needs:

a) a very high accuracy;

b) various loading histories;

c) long periods.
Importance of local strain measurements to accurately evaluate creep strain

Drained TC on saturated sedimentary soft rock
Effects of bedding error: significant on axial strains during monotonic loading, but even larger on creep strains.
Importance of various loading histories

Drained TC test on saturated undisturbed Pleistocene clay from Osaka Bay

To capture the overall picture of viscous properties;

During otherwise monotonic loading and unloading at constant strain rate;
1) changes in the strain rate;
2) sustained loading; and
3) switching between strain control and stress control.
What is the response of geomaterial?

Deviator stress, $q$
Elapsed time, $t$

Tests (1) – (6)

Different loading histories in drained TC tests

Constant strain rate during ML

?
How to predict a strain increment, $d\varepsilon$?

Generally accepted concept:

$$d\varepsilon = \text{elastic component, } d\varepsilon^e,$$

$$+ \text{ in-elastic (or irreversible) component, } d\varepsilon^{ir}$$
Elasto-plastic
- no viscosity
- no cyclic loading effect
- no ageing effect

Plastic strains:
  time-independent; and
  irreversible!

A unique stress-strain curve for all tests 1 – 6
Deviator stress, $q$

Sustained loading

Cyclic loading

Different loading histories in drained TC tests

Elasto-plastic model
- no viscosity
- no cyclic loading effect
- no ageing effect

A unique stress-strain curve for all tests 1 – 4
How to predict a strain increment, $d\varepsilon$?

Generally accepted concept:

$$d\varepsilon = \text{elastic component, } d\varepsilon^e,$$

+ in-elastic (or irreversible) component, $d\varepsilon^{ir}$

Not purely plastic (time-independent irreversible deformation!)

What is the structure that can take into account:

a) time effects; and cyclic loading effects?
Time effects: mainly viscous property and partly ageing effects

1. Background
2. Definitions of ageing effect and loading rate effect
3. Some fundamentals for modelling of viscosity property
4. Isotach viscous property
5. TESRA viscous property
6. P&N viscous property
7. Non-linear three-component model
8. Effects of particle characteristics on viscous properties
9. Creep strain and cyclic loading-induced residual strain
10. Ageing effect and its modelling
Two different components of “the time effects”

<table>
<thead>
<tr>
<th>Phenomenon</th>
<th>Mechanism/material property</th>
<th>Parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ageing effect</td>
<td>Time-dependent changes in the material property+</td>
<td>Time with the fixed origin (t&lt;sub&gt;c&lt;/sub&gt;)</td>
</tr>
<tr>
<td>Loading rate effect</td>
<td>A rate-dependent response of material due to <strong>viscous</strong> property</td>
<td>Strain rate, ( \dot{\varepsilon}^{ir} )</td>
</tr>
<tr>
<td>(creep, stress relaxation, etc.)</td>
<td></td>
<td>e.g., air-dried Sand*</td>
</tr>
</tbody>
</table>

* Positive ageing: e.g., cementation
Negative ageing: e.g., weathering

* excluding geological digenetic effects
a) Ageing effects;
defined as time-dependent changes in the intrinsic stress-strain properties, including peak strength, elasticity, yielding and viscosity. The ageing effect can be expressed as a function of the time \( t_c \) having a specifically defined

b) Loading rate effects;
defined as the rate-dependency of stress-strain behaviour due to the viscous property, typically noted by creep deformation, stress-relaxation and strain rate effects on stress-strain behaviour. The viscous property is a function of instantaneous irreversible (or visco-plastic) strain rate and other relevant parameters, not by time.
### Two different components of “the time effects”

<table>
<thead>
<tr>
<th>Phenomenon</th>
<th>Mechanism/material property</th>
<th>Parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ageing effect</td>
<td>Time-dependent changes in the material property+</td>
<td>Time with the fixed origin ($t_c$)</td>
</tr>
<tr>
<td>Apparent ageing (creep, stress relaxation, etc.)</td>
<td>A rate-dependent response of material due to <strong>viscous</strong> property</td>
<td>Strain rate, $\dot{\varepsilon}^{ir}$</td>
</tr>
<tr>
<td>e.g., cement-mixed soil</td>
<td></td>
<td>e.g., air-dried Sand**</td>
</tr>
</tbody>
</table>

* Positive ageing: e.g., cementation
Negative ageing: e.g., weathering

* excluding geological digenetic effects
Elasto-viscoplastic (shear yielding only & Isotach viscosity) - no ageing effects

Different stress-strain curves by viscosity, controlled by $\dot{\varepsilon}_a^{ir}$

The same stress-strain curves for tests 1 & 2 because of no ageing effect & no volumetric yielding

Different loading histories in drained TC tests

Constant strain rate during ML
Elasto-viscoplastic (double yielding & isotach viscosity) - no ageing effects

Different stress-strain curves by viscosity, controlled by $\dot{\varepsilon}^{ir}_a$

Different stress-strain curves in tests 1 & 2 due to volumetric creep at $q=0$ in test 2
About double yielding; two major yield locus types

High stiffness zone developed, typically, by isotropic compression

\( q = \sigma_a - \sigma' \)

\( p' = \frac{1}{3} (\sigma_a' + 2 \sigma') \)

\( q = \frac{1}{3} (\sigma_a' + 2 \sigma') \)

a) Closed type (volumetric yielding)

High stiffness zone developed by shearing, typically, at constant \( p' \)

b) Open type (shear yielding)

<table>
<thead>
<tr>
<th>Zone</th>
<th>Shear</th>
<th>Volumetric</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>unloading</td>
<td>unloading</td>
</tr>
<tr>
<td>2</td>
<td>loading</td>
<td>unloading</td>
</tr>
<tr>
<td>3</td>
<td>unloading</td>
<td>loading</td>
</tr>
<tr>
<td>4</td>
<td>loading</td>
<td>loading</td>
</tr>
</tbody>
</table>
Stress path to evaluate the yielding property of sand, first employed by Poorooshashb et al. (1967) & Poorooshashb (1971)

Which is the soil behaviour?

- Maximum stress point, $m$
- Shear yield locus
- Yield point; $y$
- Volumetric yield locus

$q$ axis

$p'$ axis
Drained TC test:
Full-automated control of stress path & loading history and accurate strain measurements

- Well-lubricated top and bottom ends
- Rectangular prismatic specimens (18 cm high x 11 cm x 11 cm) using lateral LDTs to measure lateral strains free from membrane penetration effects when changing $\sigma'_h$

Nawir et al. (2003b) S & F
Test 1: $e_0 = 0.696$

Initial stress-state:
- $\sigma_v' = 30$ kPa
- $\sigma_h' = 30$ kPa

|Axial strain rate| = $\varepsilon_0 = 0.08 \% / \text{min.}$

To maintain the viscous effects constant as much as possible, the absolute axial strain rate was always kept constant.
Shear yield point

|Axial strain rate| $\varepsilon_0 = 0.08$ %/min.

Test 1: $e_0 = 0.696$

Initial stress-state:
\[\sigma'_v = 30\, \text{kPa} \]
\[\sigma'_h = 30\, \text{kPa} \]

(Nawir et al., 2003b)

The stress-strain behaviour is not perfectly elastic even below the respective shear yield point defined for large-scale shear yielding.
The stress-strain behaviour is not perfectly elastic even below the respective shear yield point defined for large-scale shear yielding.

Saturated Toyoura sand

Test 1: 
$e = 0.696$

|Axial strain rate|$= 0.08 \%$/min. 
kept constant to maintain constant viscous effects

Nawir et al. (2003b) S & F
The strain rate (absolute value) was always kept constant to maintain the viscous effect constant.

Nawir et al. (2003b) S & F
Summary:

Nawir et al. (2003b) S & F

Some variance is due to viscous effects

Deduced shear yield loci (without viscous effects)

Measured segments of shear yield locus

Deviatoric stress, q (MPa)

Mean principal stress, $p'$ (MPa)
But, shear yielding cannot explain effects of isotropic OC history and isotropic volumetric creep.
Loose Fuji river sand, drained TC

The horizontal coordinates of points A' have been shifted so that all the stress-strain curves overlap after the start of yielding.

Tatsuoka & Molenkamp (1983)

Shear yielding cannot explain effects of isotropic OC.
Loose Fuji river sand, drained TC

Shear yielding cannot explain effects of isotropic OC.

The horizontal coordinates of points A’ have been shifted so that all the stress-strain curves overlap after the start of yielding.

Tatsuoka & Molenkamp (1983)
The volumetric yield locus cannot extend into the dilative zone (see yield locus B-B”-B’).

The two yielding types interact with each other.

Both yielding types are relevant (i.e., double hardening), but

**Volumetric yielding: dominant with soft clay**

**Shear yielding: dominant with sand & gravel**

Tatsuoka & Molenkamp (1983)
Isotopic sustained loading have significant effects on the stress-strain behaviour during subsequent drained TC loading.

(Kiyota et al., 2005; Kiyota & Tatsuoka, 2005)
Significant effects of volumetric creep at $q=0$ on the subsequent stress-strain behaviour in drained TC

Kiyota et al. (2005); Kiyota & Tatsuoka (2005), S &
Definitions of ageing and loading rate effects

Two different components of “the time effects”

Ageing effects: changes with time in the material properties (including strength, stiffness, elasticity, plasticity and viscosity)
- Positive (e.g., cementation) & negative (weathering)

Loading rate effects due to viscosity (one feature of the material properties)

<table>
<thead>
<tr>
<th>Phenomenon</th>
<th>Mechanism or Material Property</th>
<th>Parameter</th>
</tr>
</thead>
</table>
| e.g., cement-mixed soil | **Ageing effect**
| Time-dependent: Material properties change with time, e.g., cementation, weathering, etc. | Time with the fixed origin ($t_c$) |
| Loading rate effect (creep, stress relaxation, etc.) | **Apparent ageing**
| Rate-dependent: Responses of materials due to viscous property | Strain rate, $\dot{\varepsilon}^{ir}$ | e.g., air-dried sand |
Can we say that this phenomenon is due to ageing effects?
Apparent ageing effects:
stiff, nearly elastic behavior, due to the viscous properties, taking place even when any ageing effects are not involved!
Elasto-viscoplastic (double yielding & isotach viscosity) - no ageing effects

Apparent ageing effect due to viscous properties in test 3

Different loading histories in drained TC tests

Constant strain rate during ML
Some existing elasto-plastic models assume different yield loci for:

A. anisotropic (or $K_0$-) consolidation; &

I. isotropically consolidation,
not taking into account viscous effects (but not relevant).

Tatsuoka et al. (1999a) Hamburg
Even without ageing effect……

- Relatively fast ML along stress paths 1 & 2
- Relatively short drained creep at stress point S
- Relatively fast ML

Then, different high stiffness zones

Growth to a common zone by relatively long drained creep at S

Further growth by longer sustained loading

Relatively fast ML

\[ p' \]

Tatsuoka et al. (1999a) Hamburg
Reconstituted Fujinomori clay

(Fig. 2.4 of Tatsuoka et al. 1999d)
Reconstituted Fujinomori clay

Tested by Momoya, T. (1998); Tatsuoka et al. (1999d), Torino

Development of high-stiffness zone due solely to viscous properties:
- apparent ageing effect (not true ageing effect)

Deviator stress, \( q = \sigma_1 - \sigma_3 \) (kPa)

Effective mean principal stress, \( p' = (\sigma_1' + 2\sigma_3')/3 \) (kPa)

Axial strain, \( \varepsilon_v \) (%)
Undrained TC: $\dot{\varepsilon}_v = \dot{\varepsilon}_0 = 0.05\%/\text{min.}$

Anisotropic compression at $\dot{\varepsilon}_v = 0.15 \dot{\varepsilon}_0$

Reconstituted Fujinomori clay
Undrained TC

The yielding is controlled by “drained creep at $S$”, little by previous stress paths!

Only specimen 16; drained creep for two days at $S$

Tatsuoka et al. (1999d), Torino
All specimens; drained creep for two days at S

Similar high stiffness zones by “the same drained creep at S” despite different stress paths until S.

Tested by Momoya, T. (1998); Tatsuoka et al. (1999d), Torino

Reconstituted Fujinomori clay
Undrained TC
Two different components of “the time effects”

<table>
<thead>
<tr>
<th>Phenomenon</th>
<th>Mechanism/material property</th>
<th>Parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ageing effect</td>
<td>Time-dependent changes in the material property+</td>
<td>Time with the fixed origin ($t_c$)</td>
</tr>
<tr>
<td>Loading rate effect</td>
<td>A rate-dependent response of material due to viscous property</td>
<td>Strain rate, $\dot{\varepsilon}_{ir}$</td>
</tr>
<tr>
<td>e.g., cement-mixed soil (creep, stress relaxation, etc.)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

+ Positive ageing: e.g., cementation
Negative ageing: e.g., weathering

* excluding geological digenetic effects
A case history in which ageing effects is an engineering issue
A case history in which ageing effects is an engineering issue

The first (not the last, I hope) bridge abutment of cement-mixed gravel at Takada, completed March 2003.

(Tatsuoka, 2004)
The conventional RC wall structure, supporting the backfill with the earth pressure from the backfill.
Conventional type versus new type

The conventional RC wall structure, supporting the earth pressure from the backfill.

The backfill supporting the RC parapet without the earth pressure on the parapet.

(only design)  (actually constructed)
Deformation of cement-mixed gravel backfill by:
- **dead load**;
- **train loading**; and
- **seismic load**

Ageing effects should be taken into account for this issue.
Ageing effects:

Geological digenetic effects: clay $\rightarrow$ sedimentary soft rock (mudrock) $\rightarrow$ hard rock (mudrock)

Artificial: concrete, cement-mixed soil

$\sigma$  |  $\varepsilon$
---|---
$t_c = 1$
$t_c = 100$
$t_c = 1,000$

$t_c$ increasing

Aged for different periods, $t_c$

Same strain rate
With normal concrete, the strength increase rate after one year is relatively small.
With cement-mixed soil, the strength increase rate even after one year is relatively large.
The ratio between compressive strengths after 3 years and 28 days increases with a decrease in the strength.

The ratio decreases with an increase in the confining pressure and with a decrease in the water content.
Elasto-viscoplastic with ageing effect:
- no interaction between yielding and ageing effect

Different stress-strain curves between tests 1 & 2 due to ageing effect

A large high-stiffness zone from point b in test 3

The same behaviour for the same strain rate and the same ageing period in tests 2 & 3.
With ageing effect (changes in the stress-strain properties with time) ......

- Relatively fast ML along stress paths 1 & 2
- Relatively short drained creep at stress point S
- Relatively fast ML

Then, different high stiffness zones

Growth to a common zone by relatively long drained creep at S

Further growth by ageing effect

Relatively fast ML

Tatsuoka et al. (1999a) Hamburg
Development of yield locus by coupled viscous and ageing effects:

Cement-mixed poorly graded sand

(Kongsukprasert et al., 2002)
Development of high stiff quasi-elastic zone

Consolidation

Ageing

Yielding

Test                  wi            c/s  γd
A11APSC     22.378     4.349     1.231
A11BPSC     21.595     4.323     1.241
A11CPSC     21.779     4.322     1.237

Deviator stress, q, (kPa)

Shear strain, γ, (%)

(Kongsukprasert et al., 2002)
The size and shape of high stiffness zone is controlled by the ageing at a fixed point A, independent of the previous stress path.

(Kongsukprasert et al., 2002)
We could see part of the yield locus, but the whole yield locus is not known yet.
Elasto-viscoplastic with ageing effect:
- no interaction between yielding and ageing effect

Different stress-strain curves between tests 1 & 2 due to ageing effect

A large high-stiffness zone from point b in test 3

The same behaviour for the same strain rate and the same ageing period in tests 2 & 3.
Nearly the same peak strength, showing negligible interaction between yielding and ageing effect

Kaolin ($G_s = 2.65$, $D_{50} = 4.1 \mu m$, $w_L = 45.9 \%$, $I_p = 26.5$) mixed with high-early strength Portland cement (3 \% by weight)

Consolidated & saturated

Cured at $q = 0$ for 24 hours

Recured (i.e., drained creep) for 24 hours

Komoto et al. (2004)
Nearly the same peak strength, showing negligible interaction between yielding and ageing effect

Kaolin ($G_s = 2.65$, $D_{50} = 4.1 \, \mu m$, $w_L = 45.9 \, \%$, $I_p = 26.5$) mixed with high-early strength Portland cement (3 \% by weight).

Consolidated & saturated

Other similar tests show the same trend ....

Komoto et al. (2004)
Elasto-viscoplastic model with positive ageing effects: positive coupling between the effects of yielding & ageing

Even for the same strain rate and the same ageing period, a larger ultimate strength when aged longer at a higher deviator stress.
Drained TC tests to evaluate ageing effects as well as loading rate effects

 Material

 Chiba gravel ($D_{\text{max}} = 10 \text{ mm}$) mixed with of Portland cement (2.5 % of gravel) and water content 8.75 %
 Specimen: 9.5 cm x 9.5 cm x 19 cm ($\rho_d = 2 \text{ g/cm}^3$)

 Triaxial compression Apparatus

 Locally axial and lateral strain measurements with LDTs.
Compacted moist cement-mixed gravel in drained TC ($\sigma'_h = 19.7$ kPa and an axial strain rate of 0.03 %/min)

Larger peak strength by ageing longer at an anisotropic stress state!

- Aged for 60 days at $q = 0$
- Aged for 37 days at $q = 0$
- Then aged 30 days at $q = 200$ kPa

Over-shooting
High stiffness for a large stress range

Kongsukprasert & Tatsuoka (2005b), S & F

Axial strain measured with LDT (%)
Larger peak strength when aged longer at a more anisotropic stress state!

due to positively interacting effects between yielding and ageing with a shear stress!

(Kongsukprasert & Tatsuoka, 2005)
Peak strengths that increased more by longer ageing at a more anisotropic stress state.

$q_{\text{max}} - t_c$ relation when cured without a shear stress.
During very slow loading, the tangent stiffness increases and the peak strength becomes larger, both due to ageing effect.

Consolidated

Elapsed time, $t$ (days)

Deviator stress, $q$ (MPa)

Average axial strain, $\varepsilon_{a(ave)}$ (%)

Very slow ML tests

Test JS008
$t_{ini} = 8$ days

Test JS009
$t_{ini} = 8$ days

"$t$ at peak" = around 14 days

Kongsukprasert & Tatsuoka (2005), S & F
By continuing ageing effect during very slow loading, the tangent stiffness & peak strength increase.

Kongsukprasert & Tatsuoka (2005), S & F
Peculiar behaviour due to continuing ageing effect

ML at a constant $\dot{\varepsilon}$

Decrease in $\varepsilon$

Increase in $\varepsilon^*$
Kongsukprasert & Tatsuoka (2005), S & F
Significant positive interaction effects between yielding & ageing during ML at a very low strain rate!

(Kongsukprasert & Tatsuoka, 2005)
Positive interaction effect between yielding and ageing at anisotropic stress states on the compressive strength

\[ q_{\text{max}}(t_c + dt_c) = (q_{\text{max}})_o(t_c) + S \cdot dt_c + B \cdot S \cdot dt_c + \Delta q_{\text{gain}} \]

AA': peak strength for ML without curing at non-zero shear stress

\[ (q_{\text{max}})_o(t_c + dt_c) = (q_{\text{max}})_o(t_c) + S \cdot dt_c \]

\[ \Delta q_{\text{gain}} = \Delta q_0 \]

\[ B = \frac{\Delta q_{\text{gain}}}{\Delta q_0} \]

\[ A = \frac{C}{T} \]

Area C

Area T

Loading path of the concerned TC test

Start of curing

Elapsed time since the start of curing, \( t_c \)

Deviator stress, \( q \)
More positive ageing effects on $q_{\text{max}}$ when subjected to more drained creep at more anisotropic stress states

The specimen was subjected to drained creep at anisotropic stress states at the youngest age among these tests.

Fitted curve (corrected for viscosity): $B = 7.85112(A)^{5.99}$

The underlined numbers indicate the data in added to those reported by Kongsukprasert & Tatsuoka (2003)

(Kongsukprasert & Tatsuoka, 2005)
Time effects: mainly viscous property and partly ageing effects

1. Background
2. Definitions of ageing effect and loading rate effect
3. Some fundamentals for modelling of viscosity property
4. Isotach viscous property
5. TESRA viscous property
6. P&N viscous property
7. Non-linear three-component model
8. Effects of particle characteristics on viscous properties
9. Creep strain and cyclic loading-induced residual strain
10. Ageing effect and its modelling
Some fundamentals for modelling of the viscosity properties of geomaterials

Some existing models
a) Phenomenological creep models
b) Isochronous models
c) Linear elasto-visco-plasticity models
   - Maxwell
   - Voigt
   - Linear three-component model
Phenomenological creep models

Logarithmic creep laws:

1) \( \varepsilon_{ir} = \varepsilon_1 + \alpha \cdot \ln(t) \) for \( t \geq 0 \)

where \( \varepsilon_1 \) is the creep strain \( \varepsilon_{ir} \) at \( t = 1.0 \),

but, \( \varepsilon_{ir} \) becomes negatively infinitive at \( t = 0 \).
Phenomenological creep models

Logarithmic creep laws:

2) \[ \varepsilon^i_r = \varepsilon^i_t + \alpha \cdot \ln\left(\frac{t}{t_0}\right) \text{ for } t \geq t_0 \]

where \( \varepsilon^i_{t_0} \) is \( \varepsilon^i_r \) when \( t = t_0 \),

but, still \( \varepsilon^i_r \) becomes negatively infinitive at \( t = 0 \):

and the meaning of \( t_0 \) is not clear.
Phenomenological creep models

Logarithmic creep laws:

1) $\varepsilon^{ir} = \varepsilon_1 + \alpha \cdot \ln(t)$ for $t \geq 0$

   where $\varepsilon_1$ is the creep strain $\varepsilon^{ir}$ at $t=1.0$,
   $\varepsilon^{ir}$ becomes negatively infinitive at $t=0$.

2) $\varepsilon^{ir} = \varepsilon^{ir}_0 + \alpha \cdot \ln(t/t_0)$ for $t \geq t_0$

   where $\varepsilon^{ir}_0$ is $\varepsilon^{ir}$ when $t=t_0$,
   but, still $\varepsilon^{ir}$ becomes negatively infinitive at $t=0$:
   and the meaning of $t_0$ is not clear.

3) To alleviate the above problem;
   $\varepsilon^{ir} = \alpha \cdot \ln(\frac{t \cdot \dot{\varepsilon}^{ir}_0}{\alpha} + 1)$ for $t \geq 0$

   where $\dot{\varepsilon}^{ir}_0$ is the irreversible strain rate at $t=0$. 
\[ \varepsilon^{ir} = \alpha \cdot \ln\left(\frac{t \cdot \dot{\varepsilon}^{ir}_0}{\alpha} + 1\right) \]

But………………

\[ \dot{\varepsilon}^{ir}_0 \text{ at } t = 0 \]
Major problems with all of the phenomenological creep models

1) When is the origin of $t$ (where $t=0$)?

- a very serious question, which is impossible to answer,
  resulting from the use of time $t$ as a basic variable.

*Geomaterial does not have a clock nor a watch.*
It is not possible to define the moment of $t=0$ (the start of creep test) in the objective way for different previous stress histories before the start of a creep test:

$\sigma_c(t=0 \text{ in test 1})$

$\sigma_d(t=0 \text{ in tests 2 and 3})$

Stress and strain relationship for continuous loading (e.g., at a constant $\dot{\varepsilon}^{\text{ir}}$)

Test 1 ($a \rightarrow c \rightarrow d \rightarrow e$); one-step creep loading from c;
Test 2 ($a \rightarrow b \rightarrow d \rightarrow e$); the second creep loading from point d;
Test 3 ($a \rightarrow c \rightarrow f \rightarrow d \rightarrow e$); the creep loading from point d after unloading.

$t=0$ can be defined at different moments among different tests!
Major problems with the phenomenological creep models

2) cannot be applied to general cases where stress, strain and loading rate change simultaneously and arbitrarily.

3) \[ \varepsilon_{ir} = \alpha \cdot \ln \left( \frac{t \cdot \dot{\varepsilon}_{0}^{ir}}{\alpha} + 1 \right) \]; the meaning of the parameter \( \alpha \)? \( \alpha \) may depend on the stress state and strain history.
Removing the variable, time $t$, from the logarithmic creep law:

\[
\varepsilon^{ir} = \alpha \cdot \ln\left(\frac{t \cdot \dot{\varepsilon}^{ir}_0}{\alpha} + 1\right) \quad \rightarrow \quad d\varepsilon^{ir} = -\alpha \cdot d\ln(\dot{\varepsilon}^{ir})
\]

a) not including time $t$; and

b) expressed using only instantaneous variables $d\varepsilon^{ir}$ and $\dot{\varepsilon}^{ir}$, no need to define the origin of time $t$, so can be applied to general stress history in which the strain rate may change arbitrarily while the stress is changing.

c) the same with the creep equations that are obtained from;
   i) the non-linear Voigt model; and
   ii) one particular isotach model.
   (This point is explained later).
\[ \varepsilon^{ir} = \alpha \cdot \ln \left( \frac{t \cdot \dot{\varepsilon}_0^{ir}}{\alpha} + 1 \right) \] 
\[ \rightarrow \quad d \varepsilon^{ir} = -\alpha \cdot d \ln (\dot{\varepsilon}^{ir}) \] 
\[ \frac{\dot{\varepsilon}_0^{ir}}{\alpha} \cdot dt \] 
\[ d \varepsilon^{ir} = \alpha \cdot \frac{\alpha}{t \cdot \dot{\varepsilon}_0^{ir} + 1} \] 
\[ \frac{t \cdot \dot{\varepsilon}_0^{ir}}{\alpha} + 1 = \frac{\dot{\varepsilon}_0^{ir}}{\dot{\varepsilon}^{ir}} \] 
\[ dt \cdot \frac{\dot{\varepsilon}_0^{ir}}{\alpha} = -\dot{\varepsilon}_0^{ir} \cdot \frac{d \dot{\varepsilon}^{ir}}{(\dot{\varepsilon}^{ir})^2} \] 
\[ \frac{dt \cdot \dot{\varepsilon}_0^{ir}}{\alpha} \left( = \frac{d \varepsilon^{ir}}{\alpha} \right) = -\frac{d \dot{\varepsilon}^{ir}}{\dot{\varepsilon}^{ir}} \left( = -d \ln \dot{\varepsilon}^{ir} \right) \]
Isochronous model-1

1) $F_t(\sigma, \epsilon, t) = 0$ ; or $\sigma = \sigma(\epsilon, t)$
   $\sigma$ is controlled uniquely by instantaneous $\epsilon$ and $t$.

2) The same time increment $\Delta t$ elapses between given two stress states
   for any intermediate loading paths.

A number of famous isochronous models:
- Mitchell and Sign model
- $C_\alpha/C_c$ theory ($C_\alpha$: coefficient of secondary consolidation; Mesri)
The stress-strain state at the same time is unique irrespective of loading histories: i.e., the same stress-strain state is reached by different loading paths o→b; o→a→b; and o→a→c→b.

Is this correct?
The stress-strain state at the same time is unique irrespective of loading histories: i.e., the same stress-strain state \( b \) is reached by different loading paths \( o \rightarrow b; \ o \rightarrow a \rightarrow b; \) and \( o \rightarrow a \rightarrow c \rightarrow b \).

Is this correct?
When loading is restarted at a high strain rate from point b, the stress-strain state cannot go above the isochrone for \( t = t \) passing through point b (the path like b-d, which is located below the isochrone, is obtained). This behaviour results into to a notion that the creep is a deteriorating phenomenon! Is this correct?
Isochronous model-5

3) cannot explain the behaviour of a wide variety of geomaterials.

It is easy to show that any isochronous model is not relevant
When loading is restarted at a high strain rate (i.e., $10\dot{\varepsilon}_0$) from point b, the actual behaviour is like b-e, rejoining the original stress-strain relation that is obtained when loading continues at $10\dot{\varepsilon}_0$. Geomaterial can forget about what happened for a period that has elapsed, but can recognize the present state through instantaneous strain and strain rate!
Sedimentary soft rock (mudstone; Kazusa group)

Totally different periods elapsed along loading paths a-c and a-b-c, despite the same stress-strain states at point c!
Sedimentary soft rock (mudstone; Kazusa group)

A similar comparison between paths 1-3 and 1-2-3.
Two drained triaxial tests on a saturated clay:

1. Totally different time periods elapsed until point $c$, where the stress and strain values are the same, in the two tests.

2. A very similar strain rate at point $c$, where the stress and strain values are the same, in the two tests: a very important basis for constitutive model.
Two drained triaxial tests on a saturated sedimentary softrock:

1. Totally different periods elapsed until points b & c, where the stress and strain values are very similar in the respective pair of tests.

2. A very similar strain rate at point b, where the stress and strain values are the same, in the respective pair of tests.
The simplest rheology models

1) The classical Maxwell model and the classical Voigt (Kelvin) model.

- very simple, but expressing two basic features of the viscous property of geomaterials;

a) Maxwell; decomposition of strain into two sub-strain components; and

b) Voigt: decomposition of stress into two sub-stress components.
Maxwell-type models-1

1) Classical Maxwell model: \( d\varepsilon = d\varepsilon^e + d\varepsilon^v \) and \( \sigma = \sigma^e (= E \cdot \varepsilon^e) = \sigma^v (= \eta \cdot \dot{\varepsilon}^v) \)

a) Not the same with the isochronous model, nor the isotach model (explained later).

b) Always a constant creep strain rate and no stop of strain in a creep test, so this model cannot be applied to geomaterials.
Maxwell-type models-2

2) Strain-decomposition type in the generalized form:

a) \[ d\varepsilon = d\varepsilon^e + d\varepsilon^{ir} \]; relevant to geomaterials

b) \[ d\varepsilon = d\varepsilon^e + d\varepsilon^{ir} \] and \[ d\varepsilon^{ir} = d\varepsilon^p + d\varepsilon^y \]; not relevant! Why?
How to incorporate viscous effects into a constitutive model?

Extended Maxwell model (strain-additive model)

Elastic Plastic Viscous

\[ \sigma \text{ (stress)} \]
\[ \dot{\varepsilon} \text{ (strain rate)} \]

OK ?

No ! they cannot be separated.
Drained TC tests on air-dried sand

Silica sand No.4 (Dr$_c$ = 98.6%)
$\sigma'_{ht} = 400$ kPa

Effective principal stress ratio, $R$

Vertical strain, $\varepsilon_v$ (%)

ML ($\dot{\varepsilon}_v = 0.0625$ %/min)

Enomoto et al. (2006a), Roma
When ML is restarted from b, the stress-strain relation rejoins the original one d→e; i.e., “in-elastic strain during ML (a→d)” and “creep strain (a→b)” have the same origin.
Drained creep for ten hours

ML ($\dot{\varepsilon}_v = 0.0625 \%$/min)

This model wrongly predicts $b \rightarrow c$.

Enomoto et al. (2006a), Roma
Voigt-type models-1

1) Classical Voigt model:

\[
d\varepsilon = d\varepsilon^{ir} = d\varepsilon^{v} \quad \text{and} \quad \sigma = \sigma^{f} + \sigma^{v} = E \cdot \varepsilon + \eta \cdot \dot{\varepsilon}
\]

a) The reference stress and strain relationship;
   - observed when loading continues at zero strain rate.

b) Two characteristic properties;
   (1) A unique stress and strain relationship for a given strain rate;
   (2) A stepwise change in the stress by a stepwise change in the strain rate,
       - But, the stress and strain state jumps to a new one after exhibiting rigid behaviour (not soil-like behaviour).
Voigt-type models-2

Non-linear Voigt model:

\[ \sigma = \sigma^f + \sigma^v \]

\[ \sigma^v = \eta^* \cdot \ln(\dot{\epsilon} / \dot{\epsilon}_0) \]

Modified Voigt model having a non-linear dashpot
2) Non-linear Voigt model:

\[ \sigma^v = \eta^* \cdot \ln(\dot{\varepsilon} / \dot{\varepsilon}_0) \]
\[ \sigma^v = 0 \quad \text{when} \quad \dot{\varepsilon} = \dot{\varepsilon}_0 \]

→ the creep equation; \[ d \varepsilon^{ir} = -\frac{\eta^*}{E} \cdot d \ln(\dot{\varepsilon}^{ir}) \]

the same with the creep equation that are obtained from;

a) the phenomenological logarithmic creep law; and

b) one particular isotach model.

\[ \sigma = \sigma^f + \sigma^v = E \cdot \varepsilon^{ir} + \eta^* \cdot \ln(\dot{\varepsilon}^{ir} / \dot{\varepsilon}_0) \]
\[ d\sigma = E \cdot d\varepsilon^{ir} + \eta^* \cdot d \left[ \ln(\dot{\varepsilon}^{ir} / \dot{\varepsilon}_0) \right] = 0 \]
\[ E \cdot d\varepsilon^{ir} + \eta^* \cdot d \ln(\dot{\varepsilon}^{ir}) = 0 \]
Non-linear Voigt model:
\[ \sigma = \sigma^f + \sigma^v \]
\[ \sigma^v = \eta^* \cdot \ln(\dot{\varepsilon} / \dot{\varepsilon}_0) \]

But, still a stepwise change in the stress by a stepwise change in the strain rate; the stress and strain state jumps to a new stress and strain relationship, after exhibiting an infinitively large tangent stiffness.
Isotach stress-strain behaviour and models-1

1) Isotach; after Suklje (1969);
   in the classical Greek, ‘iso’ means ‘equal’ and ‘tachos’ means ‘speed’.

2) $F_{it} (\sigma, \epsilon, \dot{\epsilon}) = 0$ ; or $\sigma = \sigma(\epsilon, \dot{\epsilon})$
   $\sigma$ is a unique function of instantaneous $\epsilon$ and $\dot{\epsilon}$, independent of previous stress-strain-time history.
- This property is the same as the Voigt model.
At points a and c, the strain rate is changed suddenly; i.e., strain acceleration and deceleration is given.

The current stress and strain state is a function of instantaneous stress (the isotach behaviour); strain rates $\dot{\varepsilon}_0$, and so on, are given only at points a and c.
Isotach stress-strain behaviour and models-4

Data supporting the isotach model from;

a) Many oedometer tests on soft clays;
   e.g., Leroueil et al. (1985); Imai and Tang (1992); Imai (1995).

a) Triaxial compression tests on;
   • Haney clay (Vaid and Campanella, 1977);
   • NC Fujinomori clay;
   • stiff clay; Vallericca clay & Japanese Pleistocene clays;
   • well-graded gravel;
   • Metramo silty sand;
   • sedimentary softrocks; and
   • cement-mixed sand.
Two drained triaxial tests on a saturated clay:

1. Totally different time periods elapsed until point c, where the stress and strain values are the same, in the two tests.

2. A very similar strain rate at point c, where the stress and strain values are the same, in the two tests: a very important basis for constitutive model.
Two drained triaxial tests on a saturated sedimentary softrock:

1. Totally different periods elapsed until points b & c, where the stress and strain values are very similar in the respective pair of tests.

2. A very similar strain rate at point b, where the stress and strain values are the same, in the respective pair of tests.
A typical isotach behaviour:
CU triaxial compression test on kaolin

Simulation by the New Isotach Model

Experiment

Reference curve (in terms of total strain)

CU TC test on Kaolin
(confining pressure = 350 kPa)
Drained TC on sedimentary soft rock

(a)

Measured

Simulated

C: Drained creep for three days

*Silt-sandstone*

\[
\sigma' = 1.29 \text{MPa} \quad \dot{\varepsilon}_0 = 0.01\%/\text{min}
\]

(Hayano et al., 2001)
Isotach stress-strain behaviour and models-3

3) A unique stress and strain relationship for a given strain rate, which means:

a) By a stepwise change in the constant strain rate, the stress and strain state jumps to a new stress and strain relationship, after exhibiting a finite but high-stiffness behaviour followed by clear yielding.

b) When the strain rate is made zero, the stress suddenly changes to the value corresponding to zero strain rate (i.e., the stress relaxation takes place suddenly).

These properties a) & b) are similar to those of the Voigt models, but they are not soil-like.
ML ($\varepsilon_v = 0.0625 \, \%/\text{min}$)

Drained creep for ten hours

The three-component model can predict the measured behaviour $a \rightarrow b \rightarrow d \rightarrow e$.

Enomoto et al. (2006a), Roma
Sophisticated rheology models

1) Linear three–component model:

\[ d\varepsilon = d\varepsilon^e + d\varepsilon^{ir} \quad \text{and} \quad \sigma = E_1 \varepsilon^e = \sigma^f + \sigma^v = E_2 \varepsilon^{ir} + \eta \dot{\varepsilon}^{ir} \]

Linear elastic component \((E_2)\);

Linear-elastic component \((E_1)\):

Linear viscous component \((\eta)\);
Sophisticated rheology models-1

1) Linear three–component model:

\[ d\varepsilon = d\varepsilon^e + d\varepsilon^{ir} \quad \text{and} \quad \sigma (= E_1 \cdot \varepsilon^e) = \sigma^f + \sigma^v = E_2 \cdot \varepsilon^{ir} + \eta \cdot \dot{\varepsilon}^{ir} \]

Covering the drawbacks of the Maxwell and Voigt models, while taking advantages of the relevant properties of the two models.
Linear three–component model:

\[
d \varepsilon = d \varepsilon^e + d \varepsilon^{ir} \quad \text{and} \quad \sigma(= E_1 \cdot \varepsilon^e) = \sigma^f + \sigma^v = E_2 \cdot \varepsilon^{ir} + \eta \cdot \dot{\varepsilon}^{ir}
\]

By deleting the strain components \( \varepsilon^e \) and \( \varepsilon^{ir} \), the following basic equation is obtained:

\[
\sigma = E_2 \cdot (\varepsilon - \varepsilon^e) + \eta \cdot (\dot{\varepsilon} - \dot{\varepsilon}^e)
\]

\[
\sigma = E_2 \cdot (\varepsilon - \frac{\sigma}{E_1}) + \eta \cdot (\dot{\varepsilon} - \frac{\dot{\sigma}}{E_1})
\]

\[
\frac{\sigma}{E} + \frac{\dot{\sigma}}{E_1 \cdot E_2} - \varepsilon - \frac{\eta}{E_2} \cdot \dot{\varepsilon} = 0
\]

where \( \bar{E} = \frac{1}{\frac{1}{E_1} + \frac{1}{E_2}} \)

Linear elastic component \( (E_2) \);

Linear–elastic component \( (E_1) \):

Linear viscous component \( (\eta) \);
Linear three–component model:

1. The upper and lower bound relationships.

2. Slightly curved stress-strain relations for different constant strain rates;

3. A sudden change in the strain rate at point a; no quick rejoin to the respective original primary stress-strain curve.

The drawback 2 can be somehow alleviated by using:

\[ \sigma = \sigma^f + \sigma^v = E \cdot \varepsilon + \eta \cdot (\dot{\varepsilon})^b \]

\( (b \ll 1.0) \)
4. Start of creep test at point $a$ and start of reloading at point $b$:

a) Creep ends when the lower bound relation is reached.

b) No clear yield point before rejoining the original primary curve: this drawback can be somehow alleviated by using:

$$\sigma = \sigma^f + \sigma^v = E \cdot \varepsilon + \eta \cdot (\dot{\varepsilon})^b$$

($b \ll 1.0$)
2) Multiple linear-component models:

- Any multiple linear component model can be replaced by a non-linear three-component model.

Non-linear three-component model
(Di Benedetto et al., 2002; Tatsuoka et al., 2002)
Non-linear three-component model
(Di Benedetto et al., 2002; Tatsuoka et al., 2002)
The models we need are:

- those that can properly describe and predict;
- 1) effects of constant strain rate on the stress and strain relation;
- 2) creep deformation and stress relaxation;
- 3) post-creep & stress relaxation stress-strain-time behaviour;
- 4) effects of step change in the strain rate; and
- 6) effects of ageing (positive and negative).

Not be simple to find such a model or models.